Is There a Risk-Return Tradeoff? Evidence from High-Frequency Data[*]

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ABSTRACT

This paper examines the intertemporal relation between risk and return for the aggregate stock market using high-frequency data. We use daily realized, GARCH, implied, and range-based volatility estimators to determine the existence and significance of a risk-return tradeoff for several stock market indices. We find a positive and statistically significant relation between the conditional mean and conditional volatility of market returns at the daily level. This result is robust to alternative specifications of the volatility process, across different measures of market return and sample periods, and after controlling for macroeconomic variables associated with business cycle fluctuations. We also analyze the risk-return relationship over time using rolling regressions, and find that the strong positive relation persists throughout our sample period. The market risk measures adopted in the paper add power to the analysis by incorporating valuable information, either by taking advantage of high frequency intraday data (in the case of realized, GARCH, and range volatility) or by utilizing the market's expectation of future volatility (in the case of implied volatility index).

Is There a Risk-Return Tradeoff? Evidence from High-Frequency Data

The intertemporal relation between risk and return in the aggregate stock market has been one of the most extensively studied topics in financial economics. Most asset pricing models postulate a positive relationship between the market portfolio's expected return and risk, which is often defined by the variance or standard deviation of market returns. In his seminal paper, Merton (1973) shows that the conditional expected excess return on the aggregate stock market is a linear function of its conditional variance plus a hedging component that captures the investor's motive to hedge for future investment opportunities. Merton (1980) indicates that the hedging component becomes negligible under certain conditions, and the equilibrium conditional expected excess return on the market can be approximated by a linear function of its conditional variance:

$$E_{t-1}[R_t] = \gamma E_{t-1}[\sigma_t^2] \quad (1)$$

where γ is the representative investor's relative risk aversion parameter. This establishes the dynamic relation that investors require a larger risk premium at times when the stock market is riskier.

Despite the importance of the risk-return relationship and the apparent theoretical appeal of Merton's result, the empirical asset pricing literature has not yet reached an agreement on the existence of such a positive risk-return tradeoff for stock market indices.[1] Due to the fact that the conditional volatility of stock market returns is not observable, different approaches and specifications used by previous studies in estimating the conditional volatility are largely responsible for the conflicting empirical evidence.

This paper examines the intertemporal relation between risk and return for the aggregate stock market using high-frequency data. We incorporate valuable information from intraday market returns and construct the daily realized and GARCH volatility estimators. By sampling the return process more frequently, we improve the accuracy of the conditional volatility estimate and measure the risk-return relationship at the daily level. In addition, we consider the implied volatility index, which captures the market's forecast of future volatility, as an alternative measure of risk. We find a positive and statistically significant relation between market volatility and the expected excess return on the market. This result is

[1] See French, Schwert, and Stambaugh (1987), Campbell (1987), Chou (1988), Scruggs (1998), Baillie and DeGennaro (1990), Nelson (1991), Campbell and Hentschel (1992), Chan, Karolyi, and Stulz (1992), Chou, Engle, and Kane (1992), Glosten, Jagannathan and Runkle (1993), Whitelaw (1994), Harvey (1989, 2001), Goyal and Santa-Clara (2003), Brandt and Kang (2004), Lettau and Ludvigson (2004), Bollerslev and Zhou (2005), Ghysels, Santa-Clara and Valkanov (2005), and Ludvigson and Ng (2005).

robust to alternative specifications of the volatility process, across different market indices, sample periods, and after controlling for macroeconomic variables associated with business cycle fluctuations.

Our first measure employs intraday return data and adopts the daily realized variance measure of Andersen et al. (2003) as the ex ante conditional variance of market returns. We compute daily realized variance as the sum of squared five-minute returns for the period from 9:30 EST to 16:00 EST with an adjustment for the first-order serial correlation. As indicated by Andersen et al., the realized variance is, under suitable conditions, an unbiased estimator of the integrated variance and thus it is a canonical and natural measure of daily return volatility. This risk measure based on high-frequency intraday data has not been used in the literature to examine the relation between stock returns and stock market volatility.

Our second measure uses high-frequency market returns to estimate the ex ante conditional volatility with a GARCH model that takes into account the intraday activity patterns and systematic calendar effects prevalent in high-frequency returns. We estimate a GARCH-in-mean process with five-minute returns and compute the one-day-ahead conditional variance forecast of market returns by summing the conditional variance forecasts of five-minute returns. The daily GARCH volatility estimator introduced here incorporates more information from the intraday return process, and thus provides a more accurate measure of market risk.

An alternative approach to realized and GARCH volatility estimators is to use the implied volatilities from options.[2] The main difficulty in testing the ICAPM relation is that the conditional variance of the market is not observable and is generally filtered from past returns. However, using implied volatilities solves this problem by making conditional variance observable because it incorporates the market's forecast of future return volatility over the remaining life of the relevant option. Implied volatility is also known to be correlated with realized volatility.[3] Thus, it is of interest to use implied volatility as an alternative measure of risk.

With these three risk measures that incorporate more information than the traditional measures with daily or monthly data, we show that the intertemporal relation between stock market risk and expected returns is positive and statistically significant. More importantly, the estimated coefficients on the conditional variance and standard deviation are also consistent with economic intuition that they correspond to reasonable degrees of relative risk aversion and market price of risk, respectively.

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[2] Day and Lewis (1992) and Lamoureux and Lastrapes (1993) find that implied volatility contributes a statistically significant amount of information about volatility over the short-term forecasting horizon. Christensen and Prabhala (1998) show that while implied volatilities are biased forecasts of volatility they perform better than historical volatility models. Fleming (1998) also provides evidence that implied volatilities are more informative than daily returns when forecasting equity volatility. But Bollerslev and Zhou (2005) point out that the implied volatility may not be an unbiased estimator for the subsequent realized volatility due to the volatility risk premium it contains.

[3] Blair et al. (2001) find that the old implied volatility index (VOX) provides accurate forecasts of S&P 100 index return volatility, and is highly correlated with realized volatility measured by the sum of squared five-minute returns.

We further investigate the robustness of our results after controlling for macroeconomic variables known to forecast the stock market. As shown by Merton (1973), and subsequently pointed out by Campbell (1987), Scruggs (1998) and Guo and Whitelaw (2003), equation (1) omits the hedging component that captures the investor's motive to hedge for future investment opportunities. Thus, we include macroeconomic variables that have been shown in the literature to capture state variables that determine the investment opportunity set.[4] We find that the risk-return relation remains the same in terms of magnitude and statistical significance. In fact, the estimated coefficients on the conditional variance and standard deviation turn out to be even more significant. That is, including these control variables seems to reduce the noise in daily index returns and strengthens the positive relation between risk and return.

Given the results in Harvey (1989), Chou et al. (1992), Whitelaw (1994), and Lettau and Ludvigson (2004) that the risk-return relation may be time-varying, we estimate the dependence of expected returns on the lagged realized variance over time using rolling regressions. We show that the risk-return tradeoff is robust over time: it remains positive throughout our sample period and statistically significant most of the time.

The positive and significant risk-return relation established in our study remains intact after conducting various robustness checks: (*i*) For the results where realized volatility is used as a measure of market risk, we analyze the effect of measurement error in the realized variance and confirm our basic findings with an instrumental variable approach; (*ii*) While generating daily GARCH volatility estimator from the conditional variance forecasts of five-minute returns, the error process is likely to have a non-normal distribution. To accommodate leptokurtosis and tail-thickness in the distribution of error process, we allow the GARCH-in-mean model to follow a more flexible generalized t distribution; (*iii*) To demonstrate the robustness of the power of high frequency data, we employ the range-based volatility estimator that may be less prone to market microstructure noises.[5] The results from the range volatility turn out to be qualitatively similar to those obtained from the realized volatility measure.

One might argue that we find a significantly positive and robust results simply because the risk-return relation in our sample period is more stable than the relation in the other sample periods studied previously. To demonstrate the power of high-frequency data, we estimate three GARCH specifications commonly used in the literature using daily data and compare the results with our original GARCH estimates based on the intraday data. We show that the risk-return tradeoff estimated with daily data for

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[4] See, for example, Keim and Stambaugh (1986), Chen, Roll and Ross (1986), Campbell and Shiller (1988), Fama and French (1988), Schwert (1990), Campbell (1991), and Ferson and Harvey (1991).

[5] Several recent papers analyze the impact of market microstructure "noise" in the measurement of realized volatilities, see, for example, Bandi and Russell (2004), Hansen and Lunde (2005), and Aït-Sahalia, Mykland and Zhang (2005).

our sample period is positive, but it is insignificant for all the volatility specifications and for all the equity indexes considered in the paper. In contrast, utilizing the high-frequency data consistently results in a significantly positive risk-return tradeoff for all data sets and risk measures adopted in the paper.

An alternative explanation to our finding of a positive risk-return relationship is investor's demand for liquidity premium. Liquidity premium could contribute to such a relationship in two ways. First, the variance measures obtained from high-frequency intraday data could be inflated by the bid-ask spreads and thus become positively correlated with the illiquidity of the asset. Given that investors demand a positive liquidity premium (see, e.g., Amihud and Mendelson, 1991), this drives a positive relationship between conditional variance measures based on intraday data and expected returns. However, this explanation is unlikely to be the sole driver of our findings. One of the assets we analyze is the S&P 500 index futures. It is one of the most liquid assets available, and the effect of bid-ask spread on realized variance measures for this series is likely to be small. We also use the options implied volatility and range as alternative volatility measures and find similar results. These two volatility measures are more robust to the effect of bid-ask spread and other market microstructure issues. Second, as argued by Vayanos (2003), investors may have a time-varying liquidity preference arising from investment constraints: when volatility increases, many investors face an "implicit leverage" constraint and therefore demand a higher liquidity premium. Implicit leverage thus creates a link between volatility and expected returns in addition to the risk-return relation derived in a Merton (1980) type setting. Under this scenario, high frequency data would allow us to better capture the effect of time-varying liquidity premium on the risk-return relation.

The paper is organized as follows. Section I presents alternative measures of market risk. Section II describes our investigation of the risk-return tradeoff and relevant features of the data. Section III provides the empirical results. Section IV runs a battery of robustness checks. Section V concludes.

I. Alternative Measures of Market Risk

A. Realized Volatility

The daily realized variance (standard deviation) of market returns is traditionally measured by the squared (absolute) daily index returns, where the market return is defined as the natural logarithm of the ratio of consecutive daily closing index levels. Andersen and Bollerslev (1998a,b) indicate that these traditional measures are poor estimators of day-by-day movements in volatility, as the idiosyncratic component of daily returns is large. They demonstrate that the realized volatility measures based on intraday data provide a dramatic reduction in noise and a radical improvement in temporal stability relative to realized volatility measures based on daily returns. Andersen et al. (2003) show formally that the concept of realized variance is, according to the theory of quadratic variation and under suitable conditions, an asymptotically unbiased estimator of the integrated variance and thus it is a canonical and natural measure of daily return volatility.

Following the recent literature on integrated volatility, we use the high-frequency intraday data to construct the daily realized variance and standard deviation series. To set forth notation, let P_T denote the time T ($T \geq 0$) index level with the unit interval T corresponding to one day. The discretely observed time series process of logarithmic index returns with q observations per day, or a return horizon of $1/q$, is then defined by

$$R_{(q),T} = \ln P_T - \ln P_{T-1/q} \tag{2}$$

where $T = 1/q, 2/q, \ldots$. We calculate the daily realized variance of a market index using the intraday high-frequency (five-minute) return data as

$$VAR_T^{realized} = \sum_{i=0}^{q-1} R_{(q),T-i/q}^2 + 2\sum_{i=1}^{q-1} R_{(q),T-i/q} R_{(q),T-(i-1)/q} \tag{3}$$

where q_T is the number of five-minute intervals on day T and $R_{i,T}$ is the portfolio's logarithmic return in five-minute interval i on date T. The second term on the right-hand side adjusts for the autocorrelation in intraday returns.[6] $VAR_T^{realized}$ is the daily realized variance of the value-weighted index returns.[7] In addition to variance, we use standard deviation, $STD_T^{realized}$, of the value-weighted index returns as an alternative measure of market risk.

B. GARCH Volatility

French et al. (1987) use a GARCH-in-mean process to estimate the ex ante relation between risk premiums and volatility. Their empirical analyses with the GARCH model of Bollerslev (1986) are based on daily index return series. We also utilize a GARCH-in-mean model to estimate the ex ante relation between conditional mean and conditional volatility. In contrast to their study, we use high-frequency intraday (instead of daily) data to estimate the ex ante measures of conditional volatility. Specifically, we use a similar version of the MA(1)-GARCH(1,1) model of Andersen and Bollerslev (1998b) and parameterize the conditional variance of five-minute returns as a function of the last periods'

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[6] The same autocorrelation adjustment term in equation (3) is used by French et al. (1987), Goyal and Santa-Clara (2003) and Bali et al. (2005) when calculating monthly variances from daily data. Our results remain similar if we ignore this adjustment.

[7] Following Andersen et al. (2001), the daily realized volatility estimator for the U.S. equity market is generated using the open-to-close intraday returns because the overnight returns follow a very different dynamic process. Hence, the realized volatility may underestimate the underlying daily volatility, but it does not have any apparent influence on the statistical significance of the estimated relative risk aversion parameters.

unexpected news, the last period's variance, and a number of trigonometric terms (sine and cosine) that capture intraday seasonality and obey a strict periodicity of one day.

Let $\sigma^2_{(q),T}$ denote the conditional variance of $R_{(q),T}$ based on the information set up to time T–$1/q$. With a sampling frequency of q observations per day, the GARCH-in-mean process is given by the following equations:

$$R_{(q),T} = \alpha + \beta\sigma^2_{(q),T} + \varepsilon_{(q),T} + \theta\varepsilon_{(q),T-1/q} \quad (4)$$

$$E(\varepsilon^2_{(q),T} \mid \Omega_{T-1/q}) = \sigma^2_{(q),T} = \delta_0 + \delta_1\varepsilon^2_{(q),T-1/q} + \delta_2\sigma^2_{(q),T-1/q} + \sum_{p=1}^{4}\left(\lambda_{c,p}\cos\frac{p2\pi}{N}n + \lambda_{s,p}\sin\frac{p2\pi}{N}n\right) \quad (5)$$

where $R_{(q),T}$ is the five-minute return, $\varepsilon_{(q),T}$ is the error process and can be viewed as unexpected news or information shocks, and cos(.) and sin(.) terms capture intraday activity patterns and systematic calendar effects prevalent in high-frequency returns.[8] The first-order moving average term in equation (4) accounts for the strong serial correlation in high-frequency returns.

To investigate the relation between market risk and return at the daily level, we then compute the one-day-ahead conditional variance forecast of index returns by summing the one-step-ahead (5-minute), two-steps-ahead (10-minute),… forecasts over the next day and obtain a one-day-ahead GARCH volatility forecast. The sum of the conditional variance forecasts of five-minute returns for the period from 9:30 EST to 16:00 EST gives

$$VAR^{GARCH}_{T,f} = \sum_{i=1}^{q_T}\sigma^2_{i,T,f} \quad (6)$$

where q_T is the number of five-minute intervals on day T, $VAR^{GARCH}_{T,f}$ is the daily GARCH variance forecast for day T and $\sigma^2_{1,T,f}$, $\sigma^2_{2,T,f}$,… are the one-step-ahead, two-steps-ahead,… conditional variance forecast of five-minute returns for day T based on the information set up to day T–1. We define the daily standard deviation estimator of the GARCH-in-mean model as the square-root of $VAR^{GARCH}_{T,f}$, i.e., $STD^{GARCH}_{T,f} = \sqrt{VAR^{GARCH}_{T,f}}$.

[8] Following Andersen and Bollerslev (1998b), we assume the error process to follow a normal distribution when estimating equations (4)-(5) with maximum likelihood methodology. As will be discussed in the robustness section, we also use the generalized t distribution of McDonald and Newey (1988). The results are found to be robust across different conditional density specifications.

C. Implied Volatility

Implied volatilities are considered to be the market's forecast of the volatility of the underlying asset of an option. Specifically, the Chicago Board Options Exchange (CBOE)'s *new* VIX implied volatility index provides investors with up-to-the-minute market estimates of expected volatility by using real-time S&P500 index option bid/ask quotes. The new VIX, introduced on September 22, 2003, is obtained from the European-style S&P 500 index option prices and is a model-free implied volatility series based on the approach of Britten-Jones and Neuberger (2000).[9] As an alternative to daily realized and GARCH volatility estimators, we use the implied variance ($VAR_T^{implied}$) of the S&P 500 cash index returns and its square-root, the implied standard deviation ($STD_T^{implied}$), to examine the intertemporal relation between risk and return for the aggregate stock market.

II. Measuring Risk-Return Relationship

A. Time-Series Regressions

This paper investigates the intertemporal relation between conditional mean and conditional volatility of market returns at the daily level in the following form:

$$R_T = \alpha + \beta_1 E_{T-1}[VAR_T] + \varepsilon_T \tag{7}$$

where R_T is the daily excess return of the market portfolio and ε_T is the residual term. The coefficients α and β_1, according to Merton's (1973) ICAPM, should be zero and equal to the relative risk aversion coefficient, respectively. We measure the conditional variance, $E_{T-1}[VAR_T]$, with the lagged realized variance, $VAR_{T-1}^{realized}$, and the lagged options implied variance, $VAR_{T-1}^{implied}$.[10] We also quantify $E_{T-1}[VAR_T]$ with the one-day-ahead GARCH variance forecast, $VAR_{T,f}^{GARCH}$.

We also test an alternative form of the risk-return tradeoff at the market level, by examining whether the slope of the capital market line or the market price of risk is positive. We use the following discrete-time specification of Merton (1980):

$$R_T = \alpha + \beta_2 E_{T-1}[STD_T] + \varepsilon_T \tag{8}$$

where β_2 is the market price of risk, or the slope of the capital market line. Similar to equation (7), to measure conditional standard deviation, $E_{T-1}[STD_T]$, we use $STD_{T-1}^{realized}$ and $STD_{T-1}^{implied}$ for realized

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[9] Recent studies that employ the *new* VIX index include Bollerslev and Zhou (2005). At an earlier stage of the study, we also use the *old* implied volatility index (VOX) which is a weighted index of American implied volatilities calculated from eight near-the-money, near-to-expiry, S&P 100 call and put options based on the Black-Scholes pricing formula. The results are qualitatively similar and available upon request.

[10] A similar proxy is used by Goyal and Santa-Clara (2003) and Bali et al. (2005) for the monthly realized volatilities.

volatility and options implied volatility, respectively, and we use $STD_{T,f}^{GARCH}$ for the GARCH volatility forecast. Positive values for β_1 and β_2 imply the existence of a risk-return tradeoff, indicating that the expected returns are higher as the risk level for the market increases.

Campbell (1987) and Scruggs (1998) point out that the approximate relationship in equation (1) may be misspecified if the hedging term in ICAPM is important. To make sure that our results from estimating equations (7) and (8) are not due to model misspecification, we added to the regressions a set of control variables that have been used in the literature to capture the state variables that determine changes in the investment opportunity set:

$$R_T = \alpha + \beta_1 E_{T-1}[VAR_T] + \Gamma X_{T-1} + \varepsilon_T \quad (9)$$

$$R_T = \alpha + \beta_2 E_{T-1}[STD_T] + \Gamma X_{T-1} + \varepsilon_T \quad (10)$$

where X_{T-1} denotes the lagged macroeconomic variables proxying for business cycle fluctuations.

As presented in equations (7)-(10), we use the lagged realized and implied volatility as a proxy for the expectation of the current period's volatility. This is justified by the fact that the daily realized and implied variances and standard deviations are highly persistent.[11] For instance, the first-order serial correlations of the daily realized volatilities range from 0.52 to 0.65. The corresponding figures are 0.97 and 0.98 for the daily implied volatilities. To check the robustness of our findings, we also construct the conditional forecasts of the daily realized variance and standard deviation using an autoregressive moving average (ARMA) process. The following ARMA(p,q) specification is used to forecast daily realized volatility of market returns:

$$VAR_T^{realized} = \omega + \sum_{i=1}^{p} \rho_i VAR_{T-i}^{realized} + \sum_{i=1}^{q} \varphi_i \varepsilon_{T-i} + \varepsilon_T \quad (11)$$

$$STD_T^{realized} = \omega + \sum_{i=1}^{p} \rho_i STD_{T-i}^{reazlied} + \sum_{i=1}^{q} \varphi_i \varepsilon_{T-i} + \varepsilon_T \quad (12)$$

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[11] As argued by Stambaugh (1999), there exists a small sample bias in predictive regressions of the sort we use in this paper, because the regression disturbances are correlated with the regressors' innovations, hence the expectation of the regression disturbance conditional on the future values of regressors no longer equals zero. The small sample bias indicated by Stambaugh (1999), under the normality assumption and when the repressors follow an AR(1) process, is $-\frac{\sigma_{uv}}{\sigma_v^2}(\frac{1+3\rho}{T}) + O(1/T^2)$, where σ_v^2 is the variance of the regressors' innovations, ρ is the autoregressive coefficients of regressors, σ_{uv} is the correlation between the error terms, and *T* is the sample size. The magnitude of the bias decreases with the sample size. Since our sample consists of more than 20 years of daily data for SP500 Index Futures and CRSP Value-Weighted index and 16 years of daily data for SP500 Cash index, we expect the effect of the small sample bias on our estimates to be minor.

We use the fitted values of equations (11) and (12), denoted by $VAR_{T,f}^{realized}$ and $STD_{T,f}^{realized}$, in the ICAPM equation to examine the risk-return tradeoff.[12] Andersen et al. (2003) use a similar ARMA process with lag polynomials up to 5 days (p = 5, q = 5) to forecast the future realized volatility. The estimated values of β_1 and β_2 are robust across different lag specifications. As will be discussed, the results remain similar when we replace lagged volatilities with the fitted values from ARMA model.

B. Data

To capture the U.S. stock market returns, we use the daily logarithmic returns on the CRSP value-weighted index, S&P 500 cash index and S&P 500 index futures. Intraday return data for the S&P 500 cash index and S&P 500 index futures are obtained from the Institute for Financial Markets. The CRSP value-weighted index returns are available from July 3, 1962 to December 31, 2002. The daily realized and GARCH volatilities are constructed using intraday data for the following series and periods: S&P 500 cash index (January 3, 1986 – December 31, 2002), S&P 500 index futures (April 22, 1982 – December 31, 2002).[13]

The regular trading hours for the U.S. stock market extend from 9:30 EST to 16:00 EST. On a regular trading day, there are 78 five-minute intervals. The price of the most recent record in a given five minute interval is taken to be the price of that interval. A five-minute return is then constructed using the logarithmic price difference for a five-minute interval. The daily realized and GARCH variances of market returns are computed using equations (3) and (6), respectively. Note that the intraday data and the implied volatility data required to construct the daily risk measures only become available after the 1980s. In the following sections, we narrow our analysis to the periods where both the risk and the return measures are available.

Table I provides descriptive statistics for the daily market return and volatility measures.[14] Panel A of Table I shows that the average daily return on the U.S. stock market indices ranges from 0.037% for S&P 500 index futures to 0.057% for CRSP value-weighted index, which correspond to annualized returns of 9.3% and 14.1%. The unconditional standard deviations of daily returns are in the range of 0.90 for CRSP value-weighted index and 1.12 for S&P 500 cash index. The skewness and excess kurtosis statistics are reported for testing the distributional assumption of normality. The skewness statistics for daily returns are either negative or very close to zero. The excess kurtosis statistics are high and

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[12] Andersen et al. (2003) develop a framework for the direct modeling and forecasting of realized volatility. They find that specifying and estimating an ARMA-type process produces very accurate realized volatility forecasts and generally dominates the GARCH and related approaches such as exponentially weighted moving average and fractionally-integrated EGARCH.

[13] We also performed all of the analysis with S&P 100 cash index (January 5, 1987 – December 31, 2002) and Dow Jones (DJ) 30 cash index (January 4, 1993 – December 31, 2002). The results are similar and are available upon request.

[14] The month of the October 1987 crash period is excluded when computing the summary statistics.

significant at the 1% level, implying that the distribution of market index returns has thicker tails than the normal distribution. The first-order serial correlations for daily index returns are generally small.

Panel B of Table I presents the summary statistics of daily realized variances and standard deviations of market returns. The average daily realized variance is 0.897×10^{-4} for S&P 500 index futures and 0.912×10^{-4} for S&P 500 cash index. The average daily realized standard deviation is 0.833×10^{-2} for S&P 500 cash index and 0.836×10^{-2} for S&P 500 index futures, which corresponds to an annualized volatility of 13.22% to 13.27%. A notable point in Panel B is that the daily realized variances and standard deviations are highly persistent, as shown by the AR(1) coefficients which are in the range of 0.52 to 0.65. Not surprisingly the distribution of realized variances has much thicker tails than the distribution of realized standard deviations.

Panel C of Table I gives the descriptive statistics of daily GARCH variances and standard deviations of market returns. The average daily GARCH variance is 0.668×10^{-4} for S&P 500 cash index and 0.903×10^{-4} for S&P 500 index futures. The average daily realized standard deviation is 0.746×10^{-2} for S&P 500 cash index and 0.874×10^{-2} for S&P 500 index futures, which corresponds to an annualized volatility of 11.84% to 13.87%. The skewness and excess kurtosis statistics of GARCH volatilities are very similar to those of the realized volatilities. The daily GARCH volatilities are slightly more persistent than the daily realized volatilities. The first-order autoregressive coefficients range from 0.67 for S&P 500 index futures to 0.79 for S&P 500 index futures.

The daily implied volatility data are obtained from the CBOE for the period of January 2, 1990 to December 31, 2002. Panel D of Table I shows the summary statistics of daily implied variance and standard deviation, which are computed from the CBOE's annualized new implied volatility index as $[VIX/(100\times\sqrt{252})]^2$ and $VIX/(100\times\sqrt{252})$, respectively. The average daily implied variance is about 1.77×10^{-4} and the average daily implied standard deviation is about 1.26×10^{-2}. The daily implied volatilities are on average higher than the daily realized volatilities, and the difference can be due to the volatility risk premium and the fact that realized volatilities do not include overnight returns. The skewness and excess kurtosis statistics indicate that the distribution of daily implied volatility is skewed to the right and has thicker tails than the normal distribution. Both statistics are much lower than those for the daily realized and GARCH volatilities.

III. Empirical Results from Time-Series Regressions

A. Regression Results from Realized Volatility

Table II presents the regression results for various stock market indices. Since the CRSP value-weighted index has been widely used in previous studies to proxy for the market, we use it to capture daily returns for the aggregate stock market. In addition, we also adopt other measures of market return from the S&P 500 cash index and S&P 500 index futures. We report the regression results for each risk measure where the dependent variable is either the corresponding market returns or the CRSP value-weighted index returns.

Panel A of Table II examines the relation between daily realized volatility constructed from intraday returns on S&P 500 cash index and the daily excess return on S&P 500 cash and CRSP value-weighted index. In the first two rows, market risk is measured by the lagged realized variance. The results show that the lagged S&P 500 cash index variance is positively and significantly related to the market returns. The estimated coefficients on the lagged realized variance correspond to a relative risk aversion parameter of 5.57 for S&P 500 cash index and 4.79 for CRSP value-weighted index. The Newey-West (1987) adjusted t-statistics show that both coefficients are significant at the 1% level. The adjusted R^2 values are about 1%, which is of reasonable magnitude considering the degree of noise in daily index returns, and is consistent with previous studies. Note that the realized volatility measures underestimate the total daily return variation due to the exclusion of the overnight period. Consequently, the values of the estimated slope coefficients, although not the corresponding t-statistics, will be upward biased. This same concern applies to other results in the paper that uses realized volatility measures.

The last two rows of Panel A of Table II show the regression results where market risk is measured by the lagged realized standard deviation. The daily market price of risk is estimated to be 0.11 for S&P 500 cash index and 0.09 for CRSP value-weighted index, corresponding to monthly levels of 0.50 and 0.39, respectively, assuming 21 trading days in a month. Both coefficients are also significant at the 5% level based on the Newey-West standard errors. The adjusted R^2s are about 0.2%, smaller than those in the first two rows.

Panel B of Table II employs the high frequency intraday returns on S&P 500 index futures to calculate realized volatility and investigates its relation with the daily returns on S&P 500 index futures and CRSP value-weighted index. We observe similar patterns in the market risk-return relation. As shown in Panel B, the estimated relative risk aversion coefficient and he market price of risk parameters are positive and significant at the 5% or 1% level based on the Newey-West standard errors.

We further investigate the robustness of our findings using macroeconomic variables that proxy for business cycle fluctuations. We include the following variables in equations (9) and (10): the federal funds rate which is a closely watched barometer of the tightness of credit market conditions in the banking system and the stance of monetary policy; the default spread calculated as the difference between the yields on BAA- and AAA-rated corporate bonds; the term spread calculated as the difference between the yields on the 10-year Treasury bond and the three-month Treasury bill. The macroeconomic variables are obtained from the Federal Reserve statistics release website. As a final variable, we include the lagged return on the market to control for the serial correlation in daily returns that might spuriously affect the risk-return tradeoff.[15]

The regression results with control variables are demonstrated in Table III. Compared to the results in Table II, the estimated parameters of relative risk aversion and market price of risk remain very similar, both in magnitude and statistical significance. In fact, almost all the parameters become even more significant after incorporating business cycle variables. Inclusion of the macroeconomic variables seems to reduce the noise in daily index returns and strengthens the positive relation between risk and return. The estimated coefficients on these macroeconomic variables are, however, insignificant in most regressions. Thus, the control variables which proxy for future investment opportunities do not seem to affect the positive risk-return tradeoff.

The results presented in Tables II and III use the lagged realized volatility as a proxy for the expectation of the current period's volatility. To check the robustness of our findings, we construct the daily conditional forecasts of the realized variance and standard deviation of market returns using the ARMA model with lag polynomials of one, two, three, four, and five days. The estimated values of β_1 and β_2 are robust across different lag specifications. We provide the results with lag polynomials up to 5 days following Andersen et al. (2003). As shown in Panel A of Appendix A, the estimated coefficients on the expected conditional realized variance are about 8.15 for S&P 500 cash index and 7.15 for CRSP value-weighted index. The Newey-West adjusted t-statistics show that both coefficients are significant at the 1% level. The last two rows of Panel A show that the coefficients on the conditional standard deviation are estimated to be 0.11 for S&P 500 cash index and 0.10 for CRSP value-weighted index, and both coefficients are significant at the 5% level. Panel B also demonstrates a similar pattern in the market risk-return relation when S&P 500 index futures are used to construct daily conditional forecasts of realized volatility. The results in Appendix A indicate that the parameter estimates are similar when we replace the lagged volatilities by the conditional forecasts of daily realized volatility.

B. Regression Results from GARCH Volatility

We examine the market risk-return relation using a GARCH-in-mean model specified in equations (4)-(5) and estimated with high-frequency intraday returns. The MA(1)-GARCH(1,1) processes with sine and cosine terms are estimated with the maximum likelihood method for the S&P

[15] We could not include the dividend yields (or the dividend-price ratio) in our regressions because it is available only at the monthly frequency while our regressions are based on daily data.

500 cash index and S&P 500 index futures. The maximum likelihood parameter estimates and the t-statistics obtained from Bollerslev-Wooldridge (1992) robust standard errors are shown in Appendix B.

To examine the risk-return tradeoff at the daily level with the GARCH conditional volatility forecasts, we compute the one-day-ahead conditional variance forecast of index returns by summing the one-step-ahead (5-minute), two-steps-ahead (10-minute),... conditional variance forecasts of five-minute returns over the day. Table IV presents the regression results for the daily GARCH volatilities for various stock market indices. Panel A of Table IV examines the relation between daily GARCH volatilities of S&P 500 cash index returns and the daily excess return on the S&P 500 cash and CRSP value-weighted index. As shown in the first two rows of Panel A, there is a significantly positive relation between the GARCH variance of S&P 500 cash index and the excess return on the market. The estimated coefficients on the daily GARCH variance correspond to a relative risk aversion parameter of 11.25 for S&P 500 cash index and 9.79 for CRSP value-weighted index. The Newey-West adjusted t-statistics indicate that both coefficients are significant at the 1% level.

The last two rows of Panel A show the regression results where market risk is measured by the GARCH standard deviation forecast. The daily market price of risk is estimated to be 0.15 for S&P 500 cash index and 0.13 for CRSP value-weighted index, corresponding to the monthly level of 0.7 and 0.6, respectively. Both coefficients are significant at the 5% level with the Newey-West standard errors. Panel B of Table IV presents a similar pattern in the risk-return tradeoff when the S&P 500 index futures are used to construct daily GARCH volatilities. The results in Table IV provide strong evidence of a positive relation between risk and return for the aggregate stock market. It is worth noting that the risk-return tradeoff parameters with daily GARCH volatilities are consistent with the parameters directly estimated from the GARCH-in-mean equation with five-minute returns.[16]

To further investigate the robustness of our results with daily GARCH volatilities, we include the macroeconomic variables known to forecast the stock market and the lagged excess return in the ICAPM equation. The regression results are given in Table V. Compared to the results in Table IV, the estimated parameters of relative risk aversion and market price of risk remain very similar, both in magnitude and statistical significance. The t-statistics of these parameters are higher for all indices and risk measures. Similar to our findings with the realized volatility, inclusion of these macroeconomic variables seems to reduce the noise in daily index returns and strengthens the positive relation between GARCH volatility and the excess market return.

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[16] As shown in Table I, the magnitude of the daily GARCH variances is similar to the daily realized variances for S&P 500 index futures, but are smaller for the S&P 500 cash index. This may be due to the fact that the GARCH estimates are smoothed forecasts of expected volatility, whereas the daily realized volatilities are obtained from the realized intraday returns and are more sensitive to outlier and microstructure effects for the less liquid stocks in the S&P500 cash index.

The positive risk-return relation estimated with the GARCH models using high-frequency data is strikingly strong and robust compared to the results from previous studies with a similar focus using GARCH, but with lower frequency data. There is a long GARCH literature that has tried to identify the existence of a positive tradeoff between risk and return. These studies with different sample periods, different data sets, and different volatility specifications often lead to inconclusive results.[17] One might argue that the reason we get significantly positive and robust results is due to the possibility that the risk-return relation is more stable during our sample period, and not because of the power of intraday data. To demonstrate the contribution of high-frequency data, we estimate three GARCH specifications commonly used in the literature with daily data and compare the results with our original GARCH estimates using the high-frequency intraday data.

The results are presented in Appendix C. We estimate MA(1) GARCH-in-mean, MA(1) EGARCH-in-mean, and MA(1) GJRGARCH-in-mean specifications for the S&P 500 cash index and S&P 500 index futures.[18] The maximum likelihood estimates of the relative risk aversion parameter and the corresponding t-statistics obtained from Bollerslev-Wooldridge robust standard errors are presented in the table. The results indicate that although the risk-return tradeoff estimated with the three GARCH models and daily data for our sample period is positive, it is not statistically significant for any of the volatility specifications and the equity indexes considered in the paper.[19] In contrast, employing the high-frequency data in GARCH models has consistently resulted in a significantly positive risk-return relation at least at the 5% or 1% level for all the equity indexes.

C. Regression Results from Implied Volatility

Table VI tests whether there is a significantly positive relation between implied volatility and the expected market risk premium. Since the new VIX is an implied volatility index obtained from the S&P

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[17] French et al. (1987) estimate a GARCH-in-mean model with daily data, and find a positive and statistically significant relation for the period of 1928-1984. Chou (1988) finds a significantly positive relation with weekly data from 1962 to 1985. Chan et al. (1992) use a bivariate GARCH-in-mean model, and find an insignificant relation for the U.S. data from 1978 to 1989. Baillie and DeGennaro (1990) estimate a GARCH-in-mean model with a fat-tailed t-distribution for the period of 1970-1987, but fail to obtain a significant coefficient estimate. Nelson (1991) uses the exponential GARCH (EGARCH) model with a fat-tailed GED density, and finds a negative and significant relation for the period of 1962-1987. Campbell and Hentschel (1992) use an asymmetric GARCH model and find the relation to be positive for 1926-1951 and negative for 1952-1988, but neither is statistically significant. Glosten et al. (1993) use the monthly data from 1951 to 1989, and find a negative but insignificant relation from two asymmetric GARCH-in-mean models. Scruggs (1998) uses a bivariate EGARCH-in-mean model, and finds a positive and significant relation for the period of 1950-1994.

[18] The GJR-GARCH model introduced by Glosten et al. (1993) and the EGARCH model proposed by Nelson (1991) allow positive and negative shocks to have different impacts on the conditional variance.

[19] To accommodate leptokurtosis and tail-thickness in the distribution of the error process, we also use the generalized t distribution of McDonald and Newey (1988) in estimating the three GARCH-in-mean models with daily data. Since the qualitative results turn out to be very similar to those reported in Appendix C we do not present them in the paper. They are available upon request.

500 index options, the S&P 500 cash index is used to proxy for market returns. We also consider the widely used CRSP value-weighted index to proxy for the market.

Table VI presents the parameter estimates from regressing the one-day-ahead S&P 500 cash and CRSP value-weighted index returns on the daily implied volatility index. As shown in the first two rows of Panel A, there is a significantly positive relation between the lagged implied variance and the excess market return. The estimated coefficients on the daily implied variance correspond to a relative risk aversion parameter of 3.54 for S&P 500 cash index and 2.78 for CRSP value-weighted index. The Newey-West adjusted t-statistics indicate that both coefficients are significant at the 5% level. The last two rows of Panel A display the regression results where market risk is measured by the lagged implied standard deviation. The daily market price of risk is estimated to be 0.10 for S&P 500 cash index and 0.09 for CRSP value-weighted index. Both coefficients are significant at the 5% level based on the Newey-West standard errors.

To check the robustness of our results with daily implied volatilities, we include the macro variables and the lagged excess return in the ICAPM equation. The regression results are given in Panel B of Table VI. Compared to the results in Panel A, the estimated parameters of relative risk aversion and market price of risk are higher, both in magnitude and statistical significance. The estimated coefficients on the daily implied variance are about 8.46 for S&P 500 cash index and 8.00 for CRSP value-weighted index. The corresponding Newey-West t-statistics are about 3.06 and 2.90, indicating statistical significance at the 1% level. The estimated coefficients on the daily implied standard deviation are about 0.21 for S&P 500 cash index and 0.20 for CRSP value-weighted index. Both coefficients are significant at the 1% level based on the Newey-West standard errors. Similar to our findings with realized and GARCH volatility, including these variables seems to reduce the noise in daily returns and strengthens the positive relation between implied volatility and excess market return.

D. The risk-return tradeoff over time

Harvey (1989), Chou, Engle and Kane (1992), Whitelaw (1994), and Lettau and Ludvigson (2004) suggest that the risk-return relationship may be time-varying. Therefore, we estimate the dependence of expected returns on the lagged realized variance over time using rolling regressions. This also allows us to check whether our results are driven by a particular sample period. We focus on the returns and realized variance measures constructed from S&P 500 index futures and S&P 500 cash index. We estimate the risk-return relation specified in equation (7) for the two market indices with rolling sample periods. Due to the extreme movements of the stock market during the October 1987 crash, we exclude this month from our rolling regressions.[20]

The rolling window is set to be about half of the entire sample period for each index: 2,177 daily observations are used for S&P 500 index futures and 2,002 for S&P 500 cash index. For example, the first 2,177 daily observations of S&P 500 index futures returns and its realized variance (from 4/22/1982 to 1/3/1991) are used for estimation of the relative risk aversion parameter for 1/3/1991. The sample is then rolled forward by removing the first observation of the sample and adding one to the end, and another one-step-ahead risk-return relationship is measured. This recursive estimation procedure is repeated until December 31, 2002. The estimated relative risk aversion parameter over the rolling sample period represents the average degree of risk aversion over that sample period. Computation of the relative risk aversion parameters using a rolling window of data allows us to observe the time variation in investors' average risk aversion.

Figure 1 plots the estimated relative risk aversion parameter (β_1) over time. Panels A and B demonstrate β_1 estimates for S&P 500 index futures and S&P 500 cash index, respectively. Both indices produce similar pictures. The risk aversion parameters are always positive throughout the sample period. The parameters are between 3 and 8.7 for S&P 500 index futures, and they are in the range of 2.6 to 9.6 for S&P 500 cash index.

Figure 2 demonstrates the statistical significance of the risk-return relation over time using the t-statistics of the coefficients on lagged realized variance from rolling regressions. The 5% significance level is denoted by the horizontal line at 1.96. Impressively, the risk-return relation is significant for most of the time for the two indices. The relation is particularly strong for S&P 500 index futures. Only a handful of t-statistics out of 3,034 are less than 1.96, but they are still very close to the 1.96 line. The results from rolling regressions clearly indicate that the risk-return relation is robust over time, i.e., it is always positive throughout our sample period and statistically significant most of the time.

IV. Robustness Checks

A. Measurement Error in Realized Volatility

Several recent studies have shown that the realized volatility in general may differ from the notional volatility by a measurement error. Although the measurement errors have zero expected values and are approximately uncorrelated, nonetheless, for any given time interval there may be empirically

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[20] When we include the month of October 1987 or use a crash dummy for this month in our rolling regressions, the t-statistics of the estimated relative risk aversion parameters turn out to be even higher. The results with October 1987 and crash dummy are available upon request.

relevant deviations between realized volatility and notional volatility (see Andersen, Bollerslev, and Diebold (2004) and Andersen, Bollerslev and Meddahi (2005)). The actual size and exact distribution of the errors depend on the particular model structure assumed. Barndorff-Nielsen and Shephard (2002) provide general asymptotic distributional results for the size of the measurement error, while Meddahi (2001) presents explicit expressions for the class of eigenfunction stochastic volatility models.

To illustrate the effect of measurement error in realized variance on the risk-return tradeoff, we consider the following:

$$VAR_T^{realized} = VAR_T^{actual} + \eta_T , \tag{13}$$

where $\eta_T \sim N(0, \sigma_\eta^2)$ is the measurement error. After substituting the observed variables into the risk-return relation in equation (7), we have:

$$R_{T+1} = \beta_1 VAR_T^{realized} + (\varepsilon_{T+1} - \gamma\eta_T) \tag{14}$$

The least squares estimator, $\hat{\beta}_1^{OLS}$, from the regression of R_{T+1} directly on $VAR_T^{realized}$ is biased (even if σ_T^2, η_T, and ε_{T+1} are mutually independent):

$$\begin{aligned} p\lim \hat{\beta}_1^{OLS} &= \frac{p\lim(1/n)\sum_{T=1}^{n} VAR_T^{realized} R_{T+1}}{p\lim(1/n)\sum_{T=1}^{n}\left(VAR_T^{realized}\right)^2} = \frac{p\lim(1/n)\sum_{T=1}^{n}\left[VAR_T^{actual} + \eta_T\right]\cdot\left[\beta_1 VAR_T^{actual} + \varepsilon_{T+1}\right]}{p\lim(1/n)\sum_{T=1}^{n}\left[VAR_T^{actual} + \eta_T\right]^2} \\ &= \frac{\beta_1 Q^*}{Q^* + \sigma_\eta^2} = \frac{\beta_1}{1 + \sigma_\eta^2 / Q^*} \end{aligned} \tag{15}$$

where $Q^* = p\lim 1/n \sum_T \left(VAR_T^{actual}\right)^2$.

For illustration purpose, we apply the asymptotic distribution of measurement error derived by Barndorff-Nielsen and Shephard (2002) as:

$$\frac{VAR_T^{realized} - VAR_T^{actual}}{\sqrt{\frac{2}{3}\sum_{i=1}^{q_T} R_{i,T}^4}} = \frac{\eta_T}{\sqrt{\frac{2}{3}\sum_{i=1}^{q_T} R_{i,T}^4}} \xrightarrow{L} N(0,1) \tag{16}$$

where q_T in our empirical analysis is the number of five-minute intervals on day *T* and $R_{i,T}$ is the return in five-minute interval *i* on date *T*. We can back out a consistent estimator, $\hat{\beta}_1$, where

$$\hat{\beta}_1 = \frac{1/n \sum_{T=1}^{n} VAR_T^{realized} R_{T+1}}{1/n \sum_{T=1}^{n}\left(VAR_T^{realized}\right)^2 - \sigma_\eta^2} \tag{17}$$

Thus the presence of measurement error has an attenuation effect on the OLS estimator compared to the consistent estimator. Take the example of S&P 500 index futures series, where $\frac{2}{3n}\sum_{T=1}^{n}\sum_{i=1}^{q_t} R_{i,T}^4$ turns out to be 1.63×10^{-8}, while $\frac{1}{n}\sum_{T=1}^{n} VAR_T^2$ is 3.42×10^{-7}, and $\frac{1}{n}\sum_{T=1}^{n} VAR_T R_{T+1}$ is 1.2144×10^{-6}.[21] The consistent estimator of β_1, adjusted approximately with the asymptotic variance of the measurement error is $\hat{\beta}_1$ = 3.73, slightly higher than the OLS estimator, $\hat{\beta}_1^{OLS}$ = 3.55.

To formally account for the effect of measurement error in realized volatility in the context of risk-return tradeoff, we use an instrumental variable (IV) approach. The instrument Z_T is taken to be highly correlated with $VAR_T^{realized}$ but uncorrelated with the error terms η_T and ε_{T+1}. A natural set of candidates is the lagged values of $VAR_T^{realized}$: $VAR_{T-1}^{realized},\ldots,VAR_{T-k}^{realized}$.

We estimate the risk-return relation in equation (7) using the generalized method of moments (GMM) with the IV approach, with k = 4, and compare the coefficients with the OLS estimates where $VAR_T^{realized}$ is used directly. Note that for this section, we ignore the cross-product term shown in equation (3). As presented in Appendix D, the parameter estimates from the IV approach remain positive and statistically significant at the 5% or 1% level. Furthermore, they are very similar to the OLS estimates: the relative risk aversion parameters range from 3.55 to 7.87 with $VAR_T^{realized}$, whereas the corresponding figures are in the range of 3.92 to 8.57 with the IV approach. As reported in the last column of Appendix D, the Wald statistics from Hausman (1978) that test for the significance of the measurement error on the risk-return relation cannot reject the hypothesis that the two coefficients are the same. These results show that the risk-return relation identified in earlier sections remains intact after taking into account the measurement error in realized variance.

B. Estimating GARCH-in-Mean Model with Generalized t Distribution

To assess the impact of non-normal GARCH residuals in the risk-return tradeoff, we estimate the conditional mean and conditional variance of 5-minute returns using the MA(1) GARCH-in-mean

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[21] Given that Barndorff-Nielsen and Shephard's (2002) asymptotic distribution results are for $VAR_T = \sum_{i=1}^{q_T} R_{i,T}^2$, we focus on the squared term only and ignore the cross-product term in equation (2). There is also a distribution theory for the cross-product term separately (see Barndorff-Nielsen and Shephard (2003)). However, there is no joint asymptotic distribution known for the two terms.

model similar to that in equations (4)-(5), with the trigonometric terms omitted.[22] We model the errors in the mean equation following the generalized t (GT) distribution of McDonald and Newey (1988):

$$GT(R_t;\mu,\sigma,v,k) = \frac{C}{\sigma}\left(1+\frac{|R_t-\mu|^k}{((v-2)/k)\Phi^k\sigma^k}\right)^{-(v+1)/k}, \tag{18}$$

where $C = k/\left(2((v-2)/k)^{1/k}\,\Phi\,B(1/k,v/k)\right)$, $\Phi = (k/(v-2))^{1/k}\,B(1/k,v/k)^{0.5}\,B(3/k,(v-2)/k)^{-0.5}$, $B(.)$ is the Beta function, μ and σ are the mean and standard deviation of returns R_t, and v and k are positive parameters controlling for the height and tails of the density. The GT density in equation (18) is symmetric about its mean μ and generalizes both the Student t and GED distributions. The Student t distribution is obtained by letting $k = 2$ and the GED by letting n grow indefinitely large ($n \rightarrow \infty$). Appendix E presents the parameter estimates from the regressions of excess market returns on the daily GARCH volatility forecasts. There is a significantly positive relation between the daily GARCH variance and the excess market return. The estimated coefficients are 6.43 for S&P 500 cash index and 3.59 for S&P 500 index futures, and both are statistically significant at the 1% level.

C. Investigating Risk-Return Relationship Using Range-Based Volatility Estimators

Market microstructure noises in transaction data such as the bid-ask bounce may influence our risk measures based on the realized volatility and GARCH volatility forecasts, even though the data we use contain very liquid financial time series and thus are least subject to biases created by market microstructure effects. An alternative volatility measure that utilize information contained in the high frequency intraday data is the range-based volatility:

$$Range\,Volatility(T) = Max(\ln S_T) - Min(\ln S_T), \tag{19}$$

where $Max(\ln S_T)$ and $Min(\ln S_T)$ are in our empirical analysis the highest and lowest log stock market index levels over a sampling interval of one day. Equation (19) can be viewed as a measure of daily standard deviation. In addition to the range standard deviation, we use its square, which we refer to as the daily range variance. Alizadeh et al. (2002) and Brandt and Diebold (2003) show that the range-based volatility estimator is highly efficient, approximately Gaussian and robust to certain types of

[22] At an earlier stage of the study we have included the same trigonometric terms to the conditional variance to capture intraday seasonality, but it has been our experience while running the estimation procedures with the generalized t distribution that parameter estimation of the GARCH-in-mean models can be very tedious. Given our purpose of generating the one-day-ahead GARCH forecasts from the conditional variance forecasts of 5-minute returns, an obvious requirement is that the parameter convergence occurs reasonably quickly. In some cases, we could not get a global maximum of the likelihood function when we have included a set of sine and cosine terms. In view of these difficulties, we have restricted the specification of the conditional variance of 5-min returns to follow the standard GARCH(1,1) process.

microstructure noise such as bid-ask bounce.[23] In addition, range data are available for many assets over a long sample period.

We construct the range-based volatility estimators using the daily high and low of S&P 500 cash, S&P 100 cash, and DJ 30 cash index levels.[24] The lagged range variance and standard deviation of market returns are used to predict the one-day-ahead excess return on the market. The regression results are presented in Panels A and B of Appendix F. Similar to our findings from the daily realized, GARCH, and implied volatility estimators, the slope coefficients on the daily range volatility are positive and statistically significant at the 5% or 1% level. These results confirm the power of high frequency data in measuring market risk and indicate that the strong positive relation between risk and return holds when market volatility is quantified with daily realized variance or daily high-low ranges.

V. Conclusion

This paper provides strong evidence of a positive relation between risk and return for the aggregate stock market using high-frequency data. We construct the daily realized, GARCH, and range-based volatility estimators that incorporate valuable information from intraday returns and thus yield more precise measures of market risk. In addition, the implied volatility index that uses option prices to infer volatility expectations is considered as an alternative measure of risk. These alternative measures of market risk are employed to investigate the existence and significance of a risk-return tradeoff for several stock market indices. In support of the ICAPM, we find a positive and statistically significant relation between conditional mean and conditional volatility of market returns at the daily level. This result is robust to alternative specifications of the volatility process, across different market indices, sample periods, and after controlling for macroeconomic variables associated with business cycle fluctuations. We also analyze the risk-return tradeoff over time using rolling regressions, and find that the strong positive relation persists throughout our sample period. It is important to note that the estimated coefficients of relative risk aversion and market price of risk are not only positive and statistically significant, but they are also consistent with economic intuition and reasonable in magnitude.

What is new about our work is the focus on daily conditional volatility measures that incorporate more information, either by taking advantage of high-frequency intraday data (in the case of realized, GARCH, and range volatility) or by utilizing the market's expectation of future volatility (in the case of the implied volatility index). To compare our results with estimates using lower frequency data as in

[23] The range-based volatility estimator, however, is only unbiased under very specific distributional assumptions, i.e., under a time invariant diffusion parameter over the day. Therefore, the risk aversion parameter in equation (1) estimated with range may be biased.

[24] The data on daily high and low of S&P 500 index futures are not available, although one could approximate the range from the high frequency five-minute returns.

previous studies, we estimate three GARCH specifications commonly used in the literature to analyze the risk-return tradeoff, with daily data instead, and find the relation to be insignificant for our sample period. Thus, alternative measures of market risk adopted here add power to the analysis of the ICAPM relation and allow us to achieve more conclusive results.

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Table I. Summary Statistics of U.S. Stock Market Returns and Risk Measures

This table shows summary statistics of daily returns of the stock market indexes, and the corresponding daily realized, GARCH, and VIX implied variance and standard deviations. The table shows the mean, variance, skewness, excess kurtosis, first-order serial correlation and sample periods for all the variables. The daily market returns are from the S&P 500 cash index, S&P 500 index futures, and CRSP value-weighted index. The daily realized and GARCH variances and standard deviations are constructed from five-minute data from the S&P 500 cash index and S&P 500 index futures. Observations from October 1987 are excluded when computing these statistics.

Panel A: Daily Market Returns (10^{-2})

	Mean	*Median*	*Std*	*Skew*	*Kurt*	*AR*(1)	*Period*
SP500 Index Futures	0.043	0.052	1.112	-0.172	5.360	-0.037	4/22/1982–12/31/2002
SP500 Cash Index	0.037	0.050	1.030	-0.302	4.327	0.021	1/3/1986–12/31/2002
CRSP Value-Weighted Index	0.057	0.060	0.898	-0.213	4.315	0.071	4/22/1982–12/31/2002

Panel B: Daily Realized Volatility

Variance (10^{-4})	*Mean*	*Median*	*Std*	*Skew*	*Kurt*	*AR*(1)	*Period*
SP500 Index Futures	0.897	0.541	1.297	6.589	68.385	0.545	4/22/1982–12/31/2002
SP500 Cash Index	0.912	0.525	1.371	6.339	64.054	0.518	1/3/1986–12/31/2002
Standard Deviation (10^{-2})	*Mean*	*Median*	*Std*	*Skew*	*Kurt*	*AR*(1)	*Period*
SP500 Index Futures	0.836	0.736	0.445	2.284	9.677	0.649	4/22/1982–12/31/2002
SP500 Cash Index	0.833	0.725	0.466	2.252	8.953	0.620	1/3/1986–12/31/2002

Panel C: Daily GARCH Volatility

Variance (10^{-4})	*Mean*	*Median*	*Std*	*Skew*	*Kurt*	*AR*(1)	*Period*
SP500 Index Futures	0.903	0.619	1.039	6.453	73.739	0.666	4/22/1982–12/31/2002
SP500 Cash Index	0.668	0.422	0.771	4.728	35.149	0.735	1/3/1986–12/31/2002
Standard Deviation (10^{-2})	*Mean*	*Median*	*Std*	*Skew*	*Kurt*	*AR*(1)	*Period*
SP500 Index Futures	0.874	0.787	0.372	2.347	10.287	0.777	4/22/1982–12/31/2002
SP500 Cash Index	0.746	0.650	0.334	2.129	6.932	0.794	1/3/1986–12/31/2002

Panel D: Daily Implied Volatility

	Mean	*Median*	*Std*	*Skew*	*Kurt*	*AR*(1)	*Period*
VIX Implied Variance (10^{-4})	1.770	1.500	1.201	1.733	7.012	0.973	1/2/1990–12/31/2002
VIX Implied Std. Dev. (10^{-2})	1.264	1.225	0.411	0.842	3.610	0.981	1/2/1990–12/31/2002

Table II. Relation Between Daily Excess Market Return and Daily Realized Volatility

$VAR_{T-1}^{realized}$ and $STD_{T-1}^{realized}$ are the daily realized variance and standard deviation of market returns. The dependent variable is the one-day-ahead excess return on the S&P 500 cash index, S&P 500 index futures, or CRSP value-weighted index. In each regression, the first row gives the estimated coefficients. The second row gives the Newey-West adjusted *t*-statistics. The adjusted R^2 values are reported in the last column. *, ** indicate statistical significance (at least) at the 5% and 1% level, respectively.

Panel A: Volatility of the S&P 500 Cash Index Returns (1/3/1986 – 12/31/2002)

Variance	*Constant*	$VAR_{T-1}^{realized}$	*Adj-R^2*
S&P 500 Cash Index	-0.0002	5.5683	1.05%
	(-0.9621)	(4.4187)**	
CRSP Value-Weighted Index	0.0001	4.7901	0.99%
	(0.0466)	(3.8865)**	

Standard Deviation	*Constant*	$STD_{T-1}^{realized}$	*Adj-R^2*
S&P 500 Cash Index	-0.0006	0.1088	0.22%
	(-1.4017)	(2.1679)*	
CRSP Value-Weighted Index	-0.0003	0.0860	0.17%
	(-0.8670)	(2.0332)*	

Panel B: Volatility of the S&P 500 Index Futures Returns (4/22/1982 – 12/31/2002)

Variance	*Constant*	$VAR_{T-1}^{realized}$	*Adj-R^2*
S&P 500 Index Futures	0.0001	2.5695	2.85%
	(0.5938)	(7.5959)**	
CRSP Value-Weighted Index	0.0004	1.2983	1.20%
	(2.7162)**	(7.7213)**	

Standard Deviation	*Constant*	$STD_{T-1}^{realized}$	*Adj-R^2*
S&P 500 Index Futures	-0.0014	0.2075	1.03%
	(-2.0093)*	(2.6661)**	
CRSP Value-Weighted Index	-0.0003	0.0966	0.36%
	(-0.8590)	(2.3982)*	

Table III. Relation Between Daily Excess Market Return and Daily Realized Volatility with Control Variables

$VAR_{T-1}^{realized}$ and $STD_{T-1}^{realized}$ are the daily realized variance and standard deviation of market returns. R_{T-1} is the lagged excess return on the market. *FED* is the federal funds rate. *DEF* is the default spread calculated as the difference between the yields on BAA- and AAA-rated corporate bonds. *TERM* is the term spread calculated as the difference between the yields on the 10-year Treasury bond and the three-month Treasury bill. The dependent variable is the one-day-ahead excess return on the S&P 500 cash index, S&P 500 index futures, or CRSP value-weighted index. In each regression, the first row gives the estimated coefficients. The second row gives the Newey-West adjusted *t*-statistics. The adjusted R^2 values are reported in the last column. *, ** indicate statistical significance (at least) at the 5% and 1% level, respectively.

Panel A: Volatility of the S&P 500 Cash Index Returns (1/3/1986 – 12/31/2002)

Variance	*Constant*	$VAR_{T-1}^{realized}$	R_{T-1}	FED_{T-1}	DEF_{T-1}	$TERM_{T-1}$	*Adj-R^2*
S&P 500 Cash	-0.0008	5.9665	0.0392	0.0194	-0.0805	0.0088	1.19%
Index Return	(-0.6406)	(5.0260)**	(1.8904)	(1.0793)	(-1.0779)	(0.2826)	
CRSP Value-Weighted	-0.0008	5.5432	0.0869	0.0186	-0.0597	0.0091	1.72%
Index Return	(-0.7218)	(4.8726)**	(3.6399)**	(1.1719)	(-0.8884)	(0.3330)	

Standard Deviation	*Constant*	$STD_{T-1}^{realized}$	R_{T-1}	FED_{T-1}	DEF_{T-1}	$TERM_{T-1}$	*Adj-R^2*
S&P 500 Cash	-0.0014	0.1301	0.0319	0.0217	-0.0870	0.0149	0.31%
Index Return	(-1.1734)	(2.5261)*	(1.6962)	(1.2637)	(-1.1028)	(0.4989)	
CRSP Value-Weighted	-0.0014	0.1202	0.0778	0.0207	-0.0652	0.0146	0.75%
Index Return	(-1.2843)	(2.5762)**	(3.8033)**	(1.3753)	(-0.9228)	(0.5630)	

Panel B: Volatility of the S&P 500 Index Futures Returns (4/22/1982 – 12/31/2002)

Variance	*Constant*	$VAR_{T-1}^{realized}$	R_{T-1}	FED_{T-1}	DEF_{T-1}	$TERM_{T-1}$	*Adj-R^2*
S&P 500 Index	0.0001	2.5683	0.0043	0.0081	-0.0353	-0.0143	3.25%
Futures Return	(0.0851)	(7.4035)**	(0.1699)	(0.3556)	(-0.4363)	(-0.3521)	
CRSP Value-Weighted	-0.0001	1.3538	0.0771	0.0098	-0.0103	-0.0095	1.94%
Index Return	(-0.0539)	(8.2060)**	(3.4379)**	(0.6031)	(-0.1568)	(-0.3337)	

Standard Deviation	*Constant*	$STD_{T-1}^{realized}$	R_{T-1}	FED_{T-1}	DEF_{T-1}	$TERM_{T-1}$	*Adj-R^2*
S&P 500 Index	-0.0017	0.2240	0.0088	0.0214	-0.1490	0.0168	1.13%
Futures Return	(-1.2623)	(2.5783)**	(0.3490)	(1.0567)	(-1.6142)	(0.4776)	
CRSP Value-Weighted	-0.0011	0.1211	0.0822	0.0170	-0.0723	0.0073	0.98%
Index Return	(-1.0272)	(2.8534)**	(3.7039)**	(1.1154)	(-1.0380)	(0.2823)	

Table IV. Relation Between Daily Excess Market Return and Daily GARCH Volatility

$VAR_{T,f}^{GARCH}$ and $STD_{T,f}^{GARCH}$ are the one-day-ahead GARCH variance and standard deviation forecasts of market returns. To compute the one-day-ahead conditional variance forecast of index returns, we sum the one-step-ahead (5-minute), two-steps-ahead (10-minute),… conditional variance forecasts over the day. The dependent variable is the one-day-ahead excess return on the S&P 500 cash index, S&P 500 index futures, or CRSP value-weighted index. In each regression, the first row gives the estimated coefficients. The second row gives the Newey-West adjusted *t*-statistics. The adjusted R^2 values are reported in the last column. *, ** indicate statistical significance (at least) at the 5% and 1% level, respectively.

Panel A: Volatility of the S&P 500 Cash Index Returns (1/3/1986 – 12/31/2002)

Variance	*Constant*	$VAR_{T,f}^{GARCH}$	*Adj-R²*
S&P 500 Cash Index	-0.0004	11.253	0.71%
	(-1.3235)	(2.7165)**	
CRSP Value-Weighted Index	-0.0002	9.7901	0.69%
	(-0.6540)	(2.5768)**	

Standard Deviation	*Constant*	$STD_{T,f}^{GARCH}$	*Adj-R²*
S&P 500 Cash Index	-0.0008	0.1505	0.19%
	(-1.5284)	(2.0839)*	
CRSP Value-Weighted Index	-0.0005	0.1254	0.17%
	(-0.9867)	(2.0113)*	

Panel B: Volatility of the S&P 500 Index Futures Returns (4/22/1982 – 12/31/2002)

Variance	*Constant*	$VAR_{T,f}^{GARCH}$	*Adj-R²*
S&P 500 Index Futures	-0.0002	3.9601	2.85%
	(-0.1553)	(7.4287)**	
CRSP Value-Weighted Index	0.0003	1.9642	1.17%
	(2.2552)*	(6.7514)**	

Standard Deviation	*Constant*	$STD_{T,f}^{GARCH}$	*Adj-R²*
S&P 500 Index Futures	-0.0019	0.2633	1.12%
	(-2.2526)*	(2.7906)**	
CRSP Value-Weighted Index	-0.0006	0.1317	0.46%
	(-1.5572)	(2.8350)**	

Table V. Relation Between Daily Excess Market Return and Daily GARCH Volatility with Control Variables

$VAR_{T,f}^{GARCH}$ and $STD_{T,f}^{GARCH}$ are the one-day-ahead GARCH variance and standard deviation forecasts of market returns. R_{T-1} is the lagged excess return on the market. *FED* is the federal funds rate. *DEF* is the default spread calculated as the difference between the yields on BAA- and AAA-rated corporate bonds. *TERM* is the term spread calculated as the difference between the yields on the 10-year Treasury bond and the three-month Treasury bill. The dependent variable is the one-day-ahead excess return on the S&P 500 cash index, S&P 500 index futures, or CRSP value-weighted index. In each regression, the first row gives the estimated coefficients. The second row gives the Newey-West adjusted *t*-statistics. The adjusted R^2 values are reported in the last column. *, ** indicate statistical significance (at least) at the 5% and 1% level, respectively.

Panel A: Volatility of the S&P 500 Cash Index Returns (1/3/1986 – 12/31/2002)

Variance	*Constant*	$VAR_{T,f}^{GARCH}$	R_{T-1}	FED_{T-1}	DEF_{T-1}	$TERM_{T-1}$	*Adj-R^2*
S&P 500 Cash	-0.0021	13.563	0.0341	0.0381	-0.1296	0.0384	0.93%
Index Return	(-1.9340)	(3.5248)**	(1.7113)	(2.2899)*	(-1.6424)	(1.3486)	
CRSP Value-Weighted	-0.0021	12.650	0.0813	0.0361	-0.1057	0.0367	1.55%
Index Return	(-2.1030)*	(3.5122)**	(3.6684)**	(2.4789)*	(-1.4851)	(1.4912)	

Standard Deviation	*Constant*	$STD_{T,f}^{GARCH}$	R_{T-1}	FED_{T-1}	DEF_{T-1}	$TERM_{T-1}$	*Adj-R^2*
S&P 500 Cash	-0.0026	0.2083	0.0303	0.0324	-0.1058	0.0309	0.32%
Index Return	(-1.7915)	(2.6481)**	(1.5895)	(1.7541)	(-1.2873)	(0.9620)	
CRSP Value-Weighted	-0.0025	0.1961	0.0763	0.0309	-0.0840	0.0299	0.77%
Index Return	(-1.9651)*	(2.7826)**	(3.7280)**	(1.9211)	(-1.1457)	(1.0773)	

Panel B: Volatility of the S&P 500 Index Futures Returns (4/22/1982 – 12/31/2002)

Variance	*Constant*	$VAR_{T,f}^{GARCH}$	R_{T-1}	FED_{T-1}	DEF_{T-1}	$TERM_{T-1}$	*Adj-R^2*
S&P 500 Index	-0.0006	3.9522	0.0030	0.0104	-0.0512	-0.0101	3.25%
Futures Return	(-0.0421)	(7.1203)**	(0.1114)	(0.4517)	(-0.6290)	(-0.2461)	
CRSP Value-Weighted	-0.0002	2.0516	0.0761	0.0111	-0.0184	-0.0072	1.91%
Index Return	(-0.1464)	(6.7926)**	(3.3006)**	(0.6754)	(-0.2781)	(-0.2533)	

Standard Deviation	*Constant*	$STD_{T,f}^{GARCH}$	R_{T-1}	FED_{T-1}	DEF_{T-1}	$TERM_{T-1}$	*Adj-R^2*
S&P 500 Index	-0.0025	0.2828	0.0040	0.0268	-0.1739	0.0261	1.25%
Futures Return	(-1.8523)	(2.7509)**	(0.1551)	(1.3367)	(-1.8437)	(0.7512)	
CRSP Value-Weighted	-0.0015	0.1574	0.0793	0.0201	-0.0883	0.0128	1.08%
Index Return	(-1.4684)	(3.0885)**	(3.4642)**	(1.3056)	(-1.2272)	(0.4843)	

Table VI. Relation Between Daily Excess Market Return and Daily Implied Volatility

$VAR_{T-1}^{implied}$ and $STD_{T-1}^{implied}$ are the daily implied variance and standard deviation of market returns. R_{T-1} is the lagged excess return on the market. *FED* is the federal funds rate. *DEF* is the default spread calculated as the difference between the yields on BAA- and AAA-rated corporate bonds. *TERM* is the term spread calculated as the difference between the yields on the 10-year Treasury bond and the three-month Treasury bill. The dependent variable is the one-day-ahead excess return on the S&P 500 cash index or CRSP value-weighted index. In each regression, the first row gives the estimated coefficients. The second row gives the Newey-West adjusted *t*-statistics. The adjusted R^2 values are reported in the last column. *, ** indicate statistical significance (at least) at the 5% and 1% level, respectively.

Panel A: VIX Implied Volatility Index (1/2/1990 – 12/31/2002)

Variance	*Constant*	$VAR_{T-1}^{implied}$	*Adj-R²*
S&P 500 Cash Index	-0.0003	3.5440	0.83%
	(-1.0447)	(2.4872)*	
CRSP Value-Weighted Index	-0.0001	2.7842	0.52%
	(-0.3191)	(2.1125)*	

Standard Deviation	*Constant*	$STD_{T-1}^{implied}$	*Adj-R²*
S&P 500 Cash Index	-0.0006	0.1003	0.43%
	(-1.0577)	(2.0019)*	
CRSP Value-Weighted Index	-0.0003	0.0884	0.27%
	(-0.5053)	(1.9982)*	

Panel B: VIX Implied Volatility Index with Control Variables (1/2/1990 – 12/31/2002)

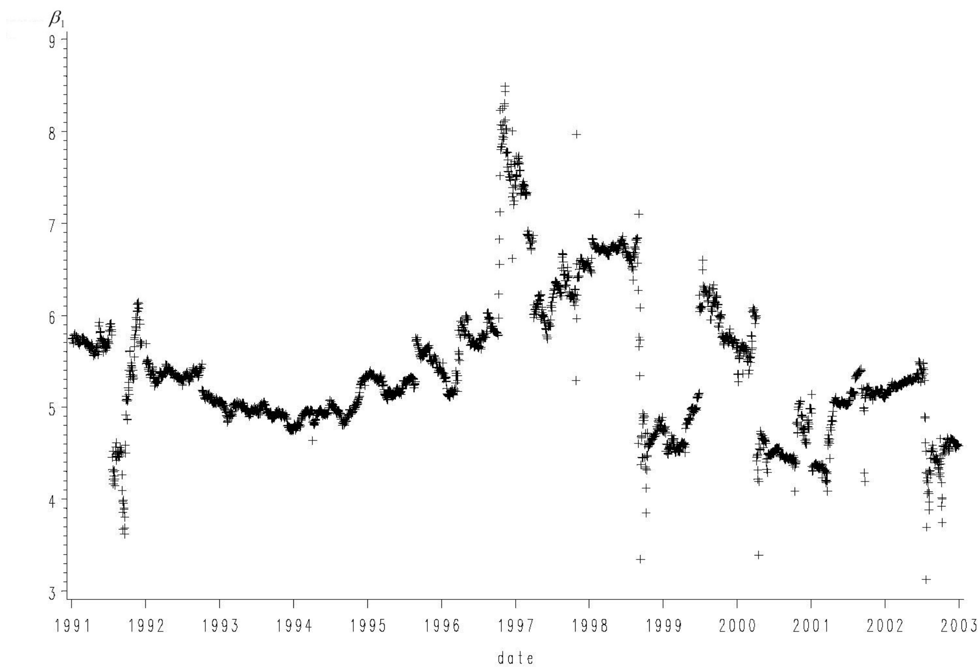
Variance	*Constant*	$VAR_{T-1}^{implied}$	R_{T-1}	FED_{T-1}	DEF_{T-1}	$TERM_{T-1}$	*Adj-R²*
S&P 500 Cash	-0.0023	8.4612	0.0338	0.0476	-0.3203	0.0822	1.12%
Index Return	(-1.6788)	(3.0579)**	(1.6032)	(2.6421)**	(-2.7681)**	(3.1190)**	
CRSP Value-Weighted	-0.0023	7.9987	0.0659	0.0458	-0.2905	0.0827	1.34%
Index Return	(-1.6951)	(2.8963)**	(3.2013)**	(2.6146)**	(-2.5614)*	(3.0797)**	

Standard Deviation	*Constant*	$STD_{T-1}^{implied}$	R_{T-1}	FED_{T-1}	DEF_{T-1}	$TERM_{T-1}$	*Adj-R²*
S&P 500 Cash	-0.0032	0.2081	0.0307	0.0414	-0.2989	0.0740	0.67%
Index Return	(-2.1107)*	(2.7683)**	(1.4568)	(2.3381)*	(-2.5591)*	(2.8191)**	
CRSP Value-Weighted	-0.0021	0.1962	0.0627	0.0399	-0.2698	0.0749	0.78%
Index Return	(-2.1042)*	(2.6351)**	(3.0446)**	(2.3260)*	(-2.3674)*	(2.8114)**	

Figure 1. The risk-return tradeoff over time

Rolling regressions with the daily excess market return as the dependent variable and the lagged daily variance as the independent variable is performed for S&P 500 index futures and S&P 500 cash index. The coefficients on VAR_{T-1}, denoted by β_1 (relative risk aversion coefficient according to ICAPM), are estimated using observations in a given time period and are plotted against the end of periods. The number of observations included in estimation is about half of the full history of the series. We use 2,177 observations for S&P 500 index futures and 2,002 for S&P 500 cash index. Observations from October 1987 are excluded when computing these statistics.

Panel A. The risk-return tradeoff for SP500 index futures returns



Panel B. The risk-return tradeoff for SP500 cash index returns

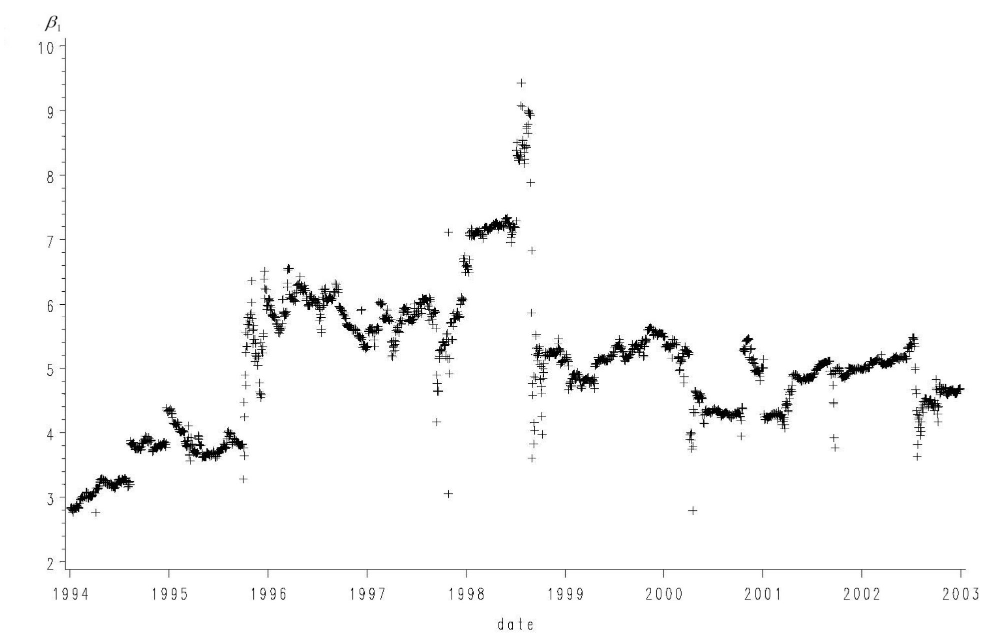
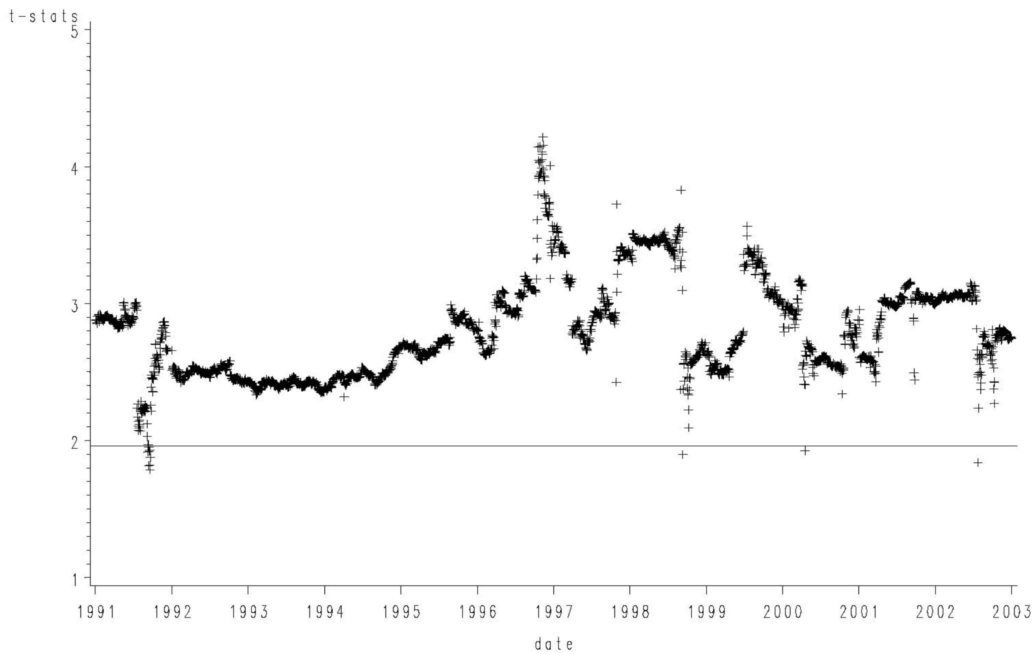


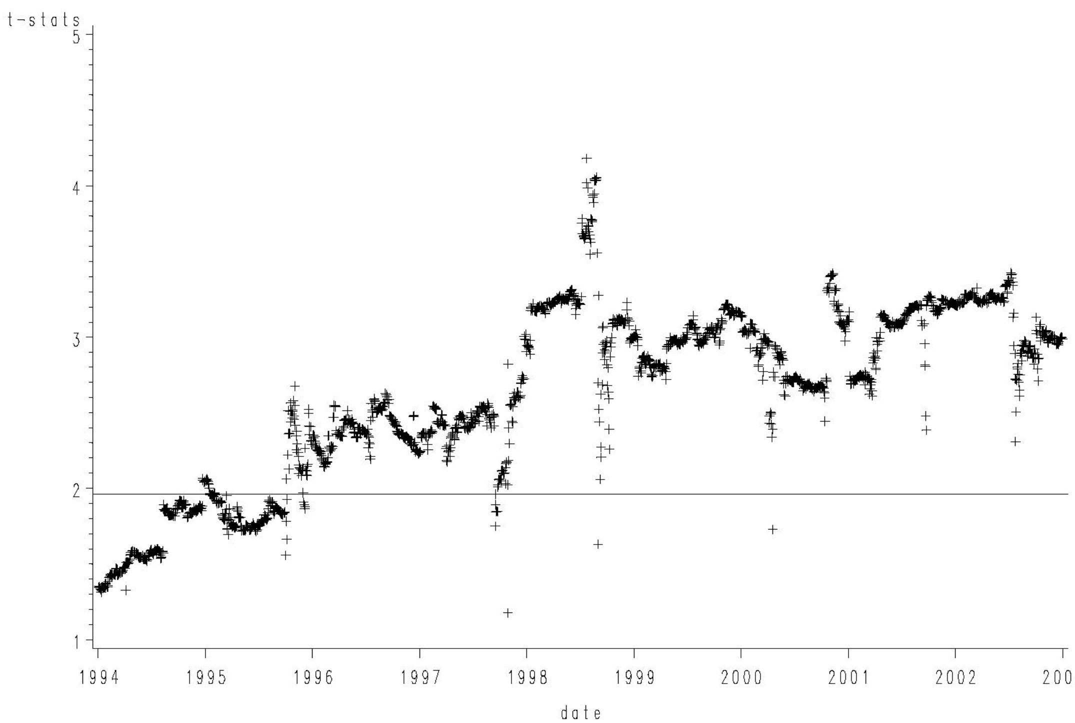
Figure 2. The significance of risk-return tradeoff over time

Rolling regressions with the daily excess market return as the dependent variable and the lagged daily variance as the independent variable is performed for S&P 500 index futures and S&P 500 cash index. The t-statistics for the coefficients of VAR_{T-1} estimated using observations in a given time period are plotted against the end of periods. The number of observations included in estimation is about half of the full history of the series. We use 2,250 observations for S&P 500 index futures and 2,000 for S&P 500 cash index. The added horizontal line represents the 5% significance level, or 1.96. Observations from October 1987 are excluded when computing these statistics.

Panel A. The risk-return tradeoff for SP500 index futures returns



Panel B. The risk-return tradeoff for SP500 cash index returns



Appendix A. Relation Between Daily Excess Market Return and Expected Realized Volatility

$VAR_{T,f}^{realized}$ and $STD_{T,f}^{realized}$ are the one-day-ahead conditional forecasts of daily realized variance and standard deviation of market returns using an ARMA(5,5) specification. The dependent variable is the excess return on the S&P 500 cash index, S&P 500 index futures, or CRSP value-weighted index. In each regression, the first row gives the estimated coefficients. The second row gives the Newey-West adjusted *t*-statistics. The adjusted R^2 values are reported in the last column. *, ** indicate statistical significance (at least) at the 5% and 1% level, respectively.

Panel A: Expected Realized Volatility of the S&P 500 Cash Index Returns (1/3/1986 – 12/31/2002)

Variance	*Constant*	$VAR_{T,f}^{realized}$	*Adj-R^2*
S&P 500 Cash Index	-0.0005 (-1.8940)	8.1510 (4.2394)**	0.85%
CRSP Value-Weighted Index	-0.0002 (-0.9717)	7.1521 (3.8803)**	0.83%

Standard Deviation	*Constant*	$STD_{T,f}^{realized}$	*Adj-R^2*
S&P 500 Cash Index	-0.0006 (-1.3320)	0.1083 (2.0798)*	0.12%
CRSP Value-Weighted Index	-0.0004 (-0.9344)	0.0986 (2.1002)*	0.13%

Panel B: Expected Realized Volatility of the S&P 500 Index Futures Returns (4/22/1982 – 12/31/2002)

Variance	*Constant*	$VAR_{T,f}^{realized}$	*Adj-R^2*
S&P 500 Index Futures	-0.0008 (-0.3480)	4.2120 (2.9734)**	1.51%
CRSP Value-Weighted Index	0.0003 (1.9699)*	1.8301 (2.1943)*	0.48%

Standard Deviation	*Constant*	$STD_{T,f}^{realized}$	*Adj-R^2*
S&P 500 Index Futures	-0.0015 (-1.9899)*	0.2399 (2.4635)*	0.67%
CRSP Value-Weighted Index	-0.0004 (-0.9210)	0.1129 (2.0193)*	0.25%

Appendix B. Maximum Likelihood Estimates of the GARCH-in-Mean Model

This table presents the maximum likelihood estimates of the GARCH-in-mean parameters based on the 5-minute returns on the S&P 500 cash index and S&P 500 index futures. The intraday data on the S&P 500 cash index cover the period from 1/3/1986 to 12/31/2002, yielding a total of 333,054 five-minute returns. The intraday data on the S&P 500 futures index span the period 4/22/1982–12/31/2002, for a total of 400,511 five-minute returns. The t-statistics obtained from Bollerslev-Wooldridge robust standard errors are given in parentheses. *, ** indicate statistical significance (at least) at the 5% and 1% level, respectively.

$$R_{(q),t} = \alpha + \beta\sigma^2_{(q),t} + \varepsilon_{(q),t} + \theta\varepsilon_{(q),t-1/q}$$

$$\sigma^2_{(q),t} = \delta_0 + \delta_1\varepsilon^2_{(q),t-1/q} + \delta_2\sigma^2_{(q),t-1/q} + \sum_{p=1}^{4}\left(\lambda_{c,p}\cos\frac{p2\pi}{N}n + \lambda_{s,p}\sin\frac{p2\pi}{N}n\right)$$

5-minute Return	S&P 500 Cash (n = 333,054)	S&P 500 Futures (n = 400,511)
α	1.21×10^{-5}	4.51×10^{-6}
	(9.6271)**	(3.0149)**
β	4.6047	5.4739
	(2.1564)*	(3.1707)**
θ	0.2243	-0.0197
	(87.251)**	(-10.862)**
δ_0	2.17×10^{-4}	1.04×10^{-4}
	(13.520)**	(32.851)**
δ_1	0.1152	0.0744
	(17.027)**	(49.616)**
δ_2	0.8623	0.9178
	(106.53)**	(665.20)**
$\lambda_{c,1}$	-1.83×10^{-4}	-1.70×10^{-4}
	(-10.593)**	(-51.054)**
$\lambda_{c,2}$	-1.57×10^{-4}	-4.36×10^{-5}
	(-4.5796)**	(-7.6342)**
$\lambda_{c,3}$	-1.25×10^{-4}	2.26×10^{-5}
	(-2.9365)**	(2.9417)**
$\lambda_{c,4}$	-7.23×10^{-5}	3.76×10^{-5}
	(-2.5651)**	(4.1208)**
$\lambda_{s,1}$	2.66×10^{-4}	4.53×10^{-5}
	(13.767)**	(10.454)**
$\lambda_{s,2}$	2.22×10^{-4}	6.78×10^{-6}
	(17.037)**	(2.1583)*
$\lambda_{s,3}$	1.95×10^{-4}	6.51×10^{-5}
	(16.514)**	(7.4227)**
$\lambda_{s,4}$	9.82×10^{-5}	5.31×10^{-5}
	(6.0010)**	(5.6662)**

Appendix C. Daily Risk-Return Tradeoff Based on Daily GARCH-in-Mean Estimates

This table presents the maximum likelihood estimates of the relative risk aversion parameter (β) based on the MA(1) GARCH-in-mean, MA(1) EGARCH-in-mean, and MA(1) GJRGARCH-in-mean models. The daily data on the S&P 500 cash index cover the period from 1/3/1986 to 12/31/2002, yielding a total of 4,290 returns. The daily data on the S&P 500 index futures span the period 4/22/1982–12/31/2002, for a total of 5,232 returns. The t-statistics obtained from Bollerslev-Wooldridge robust standard errors are given in parentheses.

$$R_t = \alpha + \beta\sigma_t^2 + \varepsilon_t + \theta\varepsilon_{t-1}$$

GARCH: $$\sigma_t^2 = \delta_0 + \delta_1\varepsilon_{t-1}^2 + \delta_2\sigma_{t-1}^2$$

EGARCH: $$\ln\sigma_t^2 = \delta_0 + \delta_1\left[\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| - \sqrt{\frac{2}{\pi}}\right] + \delta_2\ln\sigma_{t-1}^2 + \delta_3\left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right)$$

GJR-GARCH: $$\sigma_t^2 = \delta_0 + \delta_1\varepsilon_{t-1}^2 + \delta_2\sigma_{t-1}^2 + \delta_3 S_{t-1}^-\varepsilon_{t-1}^2$$

$S_{t-1}^- = 1$ for $\varepsilon_{t-1} < 0$ and $S_{t-1}^- = 0$ otherwise

	GARCH-in-Mean	EGARCH-in-Mean	GJRGARCH-in-Mean
S&P 500 Cash Index	3.4686	2.2556	1.9375
	(1.5553)	(1.1012)	(0.9153)
S&P 500 Index Futures	3.1480	2.5069	1.5993
	(1.5003)	(1.6312)	(0.9897)

Appendix D. Measurement Error in Realized Volatility

This table presents the regression results for $R_T = \alpha + \beta VAR_{T-1}^{realized} + \varepsilon_T$, where β is estimated either using the ordinary least square regression or using an instrumental variable approach with generalized method of moments (GMM). $VAR_{T-1}^{realized}$ is the lagged daily realized variance defined as the sum of squared five-minute returns. The instruments are $VAR_{T-2}^{realized}$, $VAR_{T-3}^{realized}$, $VAR_{T-4}^{realized}$, and $VAR_{T-5}^{realized}$. The dependent variable is the excess return on the S&P 500 cash index or S&P 500 index futures. The first row gives the estimated coefficients (α, β). The second row gives the Newey-West adjusted *t*-statistics. The last column reports the Wald statistics and the corresponding *p*-values in parentheses from Hausman test. *, ** indicate statistical significance (at least) at the 5% and 1% level, respectively.

	OLS Regression		**IV Approach with GMM**		
	α	β	α	β	$Wald_{Hausman}$
S&P 500 Cash Index	-0.0003	8.7230	-0.0001	7.8650	0.6457
	(-0.9958)	(2.8404)**	(-0.4922)	(2.1669)*	(0.4216)
S&P 500 Index Futures	0.0001	3.5497	0.0001	3.9167	1.0111
	(0.0290)	(7.1062)**	(0.1912)	(3.7011)**	(0.3146)

Appendix E. Daily Risk-Return Tradeoff Based on the Generalized t Distribution

$VAR_{T,f}^{GARCH}$ is the one-day-ahead GARCH variance forecasts of market returns. To compute the one-day-ahead conditional variance forecast of index returns, we sum the one-step-ahead (5-minute), two-steps-ahead (10-minute),… variance forecasts over the day obtained from the GARCH-in-mean model estimated with the generalized t distribution. The dependent variable is the one-day-ahead excess return on the S&P 500 cash index or S&P 500 index futures. In each regression, the first row gives the estimated coefficients. The second row gives the Newey-West adjusted *t*-statistics. The adjusted R^2 values are reported in the last column. *, ** indicate statistical significance (at least) at the 5% and 1% level, respectively.

Variance	*Constant*	$VAR_{T,f}^{GARCH}$	*Adj-*R^2
S&P 500 Cash Index	-0.0003	6.4305	0.71%
	(-1.0635)	(2.6811)**	
S&P 500 Index Futures	-0.0001	3.5918	2.87%
	(-0.0639)	(7.4907)**	

Appendix F. Relation Between Daily Excess Market Return and Daily Range Volatility

VAR_{T-1}^{range} and STD_{T-1}^{range} are the daily range variance and standard deviation constructed from the daily high and low prices. The dependent variable is the one-day-ahead excess return on the S&P 500 cash index, S&P 100 cash index, and DJ 30 cash index. In each regression, the first row gives the estimated coefficients. The second row gives the Newey-West adjusted *t*-statistics. The adjusted R^2 values are reported in the last column. *, ** indicate statistical significance (at least) at the 5% and 1% level, respectively.

Panel A: Range Variance of Stock Market Returns

Variance	*Constant*	VAR_{T-1}^{range}	*Adj-*R^2
S&P 500 Cash Index	0.0001 (0.4222)	1.0568 (5.3086)**	0.75%
S&P 100 Cash Index	-0.0001 (-0.5338)	1.3108 (7.5573)**	1.29%
DJ 30 Cash Index	-0.0002 (-0.5298)	0.7201 (2.1258)*	0.15%

Panel B: Range Standard Deviation of Stock Market Returns

Variance	*Constant*	STD_{T-1}^{range}	*Adj-*R^2
S&P 500 Cash Index	-0.0004 (-1.2742)	0.0558 (2.9729)**	0.18%
S&P 100 Cash Index	-0.0007 (-1.4705)	0.0702 (2.1385)*	0.30%
DJ 30 Cash Index	-0.0004 (-0.7674)	0.0473 (2.0124)*	0.11%