Spatial Market Arbitrage and Threshold Cointegration

Peter S. Sephton

Goodwin and Piggott reported that corn and soybean prices in spatially separated markets in North Carolina exhibited threshold cointegration and that commodity prices in different markets may persistently diverge. Here, a multivariate approach is used to test for threshold cointegration and nonlinear cointegration. The results suggest that departures from the law of one price do not persist indefinitely.

Key words: arbitrage, market integration, threshold cointegration, transactions costs.

Goodwin and Piggott (GP) examined daily corn and soybean prices across spatially separated markets in North Carolina and reported that bivariate pairings of prices were threshold-cointegrated. GP used a two-threshold model in which price differences across markets could persist indefinitely. They concluded that empirical models should consider threshold effects in order to adequately characterize spatially integrated markets.

The purpose of this article is to extend the GP analysis in several directions. GP employed the techniques of Martens, Kofman, and Vorst which involve a multistepped search for thresholds in the residuals of a cointegrating regression. This single-equation search process is subject to potential misspecification, so it is interesting to determine whether the direct multivariate test of threshold cointegration of Hansen and Seo provides confirmatory evidence. In addition, GP allowed two thresholds to affect each commodity pair without directly testing for the number of thresholds. The second extension uses Hansen tests to determine the number of thresholds. Two thresholds imply no-arbitrage bounds that allow persistent departures from the law of one price. One threshold is consistent with the traditional view that price differentials are fleeting as arbitrageurs drive excess returns to zero. The final extension recognizes that spatially integrated markets might follow a nonlinear relationship that depends on transactions costs. Sufficiently "large" deviations from the law of one price should lead to a reduced price dispersion. A direct test of nonlinear cointegration demonstrates that price differences tend to be resolved over time.

Background

Balke and Fomby presented one of the first descriptions of threshold cointegration, but the closely related concept of nonlinear cointegration had been introduced previously by a number of authors, including Bierens.[1] When two or more variables are drawn together over time by an equilibrating relationship, the series are cointegrated. The initial concept of cointegration assumes this relationship is linear and stable over time. Threshold cointegration allows the equilibrating relationship to change if the series exhibit different behavior beyond a threshold.

A complementary strategy involves allowing the equilibrating relationship to be nonlinear. While the law of one price suggests that the relationship between commodity prices in spatially separated markets should be linear and follow something close to a 45° line, it is not unreasonable to suspect that transactions costs can be a potential source of nonlinearity. For example, Roehner suggests that the expected price difference between markets is a function of transactions costs, the price elasticities of demand and supply, and (possibly) time-dependent heteroskedasticity. A nonlinear specification encompasses the case

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[1] See Hallman, Granger and Hallman, Sephton and the references therein for examples.

of a linear relationship. Bierens provides nonparametric, model-free tests of nonlinear cointegration that are similar to the Johansen and Juselius approach to testing for linear cointegration.[2]

GP reported that bivariate pairs of commodity prices were linearly cointegrated, using both single equation (Engle and Granger) and multiple equation (Johansen and Juselius) methods. Extending their results to establish whether spatially separated prices are nonlinearly cointegrated is the objective of the current analysis.

Empirical Results

GP examined bivariate cointegrating regressions, comparing corn and soybean prices in three markets against a fourth market which exhibited the greatest volume. GP argued that transactions costs of moving grain between markets are not negligible, and could lead to persistent deviations in prices across locations.

This analysis uses the same data as GP. GP report that the natural logarithm of each series is $I(1)$. This result was confirmed using traditional Dickey-Fuller unit root tests. The stationarity tests of Leybourne and McCabe, in which the null hypothesis is that each series is $I(0)$, and the GPH test for fractional cointegration, where the null hypothesis is that each series is $I(d)$, $d < 1$, indicate that each series can be assumed to be integrated of order one. None of the series exhibited evidence of deterministic trends on the basis of these test results.

GP concluded each bivariate commodity pair was cointegrated, except perhaps Kinston and Williamston corn prices. There appears to be several outliers in these two series, so following Tsay, several extreme values (six positive and six negative) were replaced by the average of their two nearest neighbors and the analysis was repeated. The results were qualitatively the same as reported by GP.

The null hypothesis of the Engle and Granger test is that of noncointegration: that is, the cointegrating residual contains a unit root. Many authors have demonstrated that testing the null of cointegration can lead to vastly different inferences than those drawn from unit root tests,[3] so it is of interest to examine these cointegrating residuals for direct evidence of cointegration. The stationarity test of Leybourne and McCabe was used to examine the cointegrating residuals for evidence of stationarity. This test has been shown to work well relative to the test of Kwiatkowsi, Phillips, Schmidt, and Shin, as it converges in probability at a much faster rate. In addition, it provides for the presence of autoregressive moving average (ARMA) terms in the testing process so that the results are not contaminated by omitted dynamics. Test results (available from the author upon request) are consistent with the findings reported by GP, with the exception of the cointegrating residuals from soybean markets in Greenville-Fayetteville and Kinston-Fayetteville.

How Many Thresholds?

GP adopted the Martens, Kofman, and Vorst approach of estimating threshold models. This specification has two thresholds, or three "states" in which the nature of the cointegrating relationship changes. Arbitrage opportunities, in and out of each location, are based on whether the price difference is above or below the largest or smallest threshold. Bi-directional arbitrage could also occur when a price differential reaches a single threshold, with positive differentials leading to increased sales pressure in the higher-priced market (as well as increases in demand in the lower-priced market) and negative differentials leading to the opposite behavior.

Hansen provides a series of tests to determine the number of regimes in self-exciting threshold autoregressive (SETAR) models.[4] This specification allows a variable to contain many regimes based on whether the series is above or below knots, or thresholds. A SETAR(1) model is simply a linear autoregression, while a SETAR(2) model contains one threshold, with the series behaving differently depending on whether it is above or below the threshold. When applied to the change in the cointegrating residuals (which GP used in their search for threshold effects), these diagnostics help to provide insight into the number of thresholds in the cointegrating relationships.

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[2] Coakley and Fuertes provide an accessible description of the Bierens nonparametric cointegration test.

[3] See Carrion-I-Silvestre, Sanso-I-Rossello, and Ortunio for examples.

[4] A SETAR(1) model is simply a linear autoregression without thresholds. Following Hansen, if Y_t is a univariate time series and X_{t-1} denotes a vector containing ones and lagged values of the time series, a SETAR(m) specification takes the form $Y_t = \alpha_1' X_{t-1} I_{1t}(\gamma, d) + \cdots + \alpha_m' X_{t-1} I_{mt}(\gamma, d) + \varepsilon_t$ where $I(\cdot)$ is an indicator function, d denotes the delay parameter, and the parameters contained in the vector γ are the thresholds.

Table 1. Probability Values Associated with Tests Regarding the Number of Thresholds

	SETAR(1) vs SETAR(2)		SETAR(1) vs SETAR(3)		SETAR(2) vs SETAR(3)		
Crop Locations	PV-Ho	PV-He	PV-Ho	PV-He	PV-H	PV-He1	PV-He2
Corn							
Candor-Williamston	0.14	0.08	0.002	0.069	0.09	0.08	0.41
Cofield-Williamston	0.18	0.07	0.000	0.057	0.12	0.09	0.38
Kinston-Williamston	0.61	0.15	0.404	0.553	0.99	0.99	0.99
Outlier adjusted data	0	0.001	0	0.002	0.95	0.68	0.63
Soybeans							
Raleigh-Fayetteville	0.64	0.42	0.772	0.831	0.99	0.99	0.99
Greenville-Fayetteville	0.75	0.59	0.653	0.795	0.88	0.78	0.98
Kinston-Fayetteville	0.16	0.05	0.000	0.047	0.73	0.63	0.93

Note: PV-Ho denotes the probability value assuming homoskedastic errors while PV-He denotes the probability value assuming heteroskedastic errors. PV-He1 denotes regime specific heteroskedasticity while PV-He2 denotes general heteroskedasticity. The lag lengths in the Hansen tests were set at one and the test was on the change in the cointegrating residuals with the regressor the commodity price in one of the three markets and the regressands a constant and the commodity price in the market with the largest volume (Williamston for corn and Fayetteville for soybeans).

Table 1 presents test results of various SETAR specifications and their parametric bootstrapped probability values (based on 2000 replications). The tests examine the hypothesis of a SETAR(1) model against the alternative of either SETAR(2) or SETAR(3) models as well as the null of the SETAR(2) model against the alternative of the SETAR(3) specification. Rejecting the null of a SETAR(1) model in favor of a SETAR(2) model requires that the SETAR(1) model is tested against the SETAR(3) specification to determine whether it dominates SETAR(1). That is, if there is evidence of one threshold, one needs to determine if there might be two thresholds. The SETAR(2) model is then tested against the SETAR(3). This assists in determining the number of threshold effects in the data.

When allowance is made for heteroskedasticity, tests of the null of a SETAR(1) model against the alternative of SETAR(2) are rejected at the 10% level for corn markets in Cofield, Candor, and Kinston (using the outlier-adjusted data) relative to Williamston, and for soybeans in Kinston relative to Fayetteville. These results are supported when tests of the SETAR(1) model are made against the alternative of SETAR(3), so in these markets it appears there is at least one threshold. When the SETAR(2) specification is tested against the SETAR(3) alternative, all bivariate pairs uniformly fail to reject the SETAR(2) model when general heteroskedasticity is allowed. Hence, it appears that in the case of these markets, there is at most one threshold affecting the adjustment back to equilibrium.

Soybean markets in Raleigh, Greenville, and Fayetteville exhibit little evidence of threshold effects. These results are quite different from those reported by GP using Tsay's test for thresholds. This may be, in part, because the Hansen tests consider heteroskedastic errors which can have a significant impact on inference. Taken in conjunction with the results reported previously, the double threshold model used by GP may be too rich in its design. A no-arbitrage band in which spatial price differences can persist may not fully capture the processes generating the data. A single threshold consistent with the law of one price appears to exist in corn markets in Cofield, Candor, Kinston and Williamston, and in soybean markets in Kinston and Fayetteville.[5]

Threshold Cointegration

The Hansen and Seo test of threshold cointegration is based on a vector error correction model. Let $\boldsymbol{x_t}$ be a p-dimensional $I(1)$ time series cointegrated with one $p \times 1$ cointegrating vector β. Let $w_t(\beta) = \beta' \boldsymbol{x}_t$ denote the $I(0)$ error correction term. A two-regime threshold cointegration model takes the form

$$(1) \qquad \Delta x_t = \begin{cases} A_1' X_{t-1}(\beta) + u_t & \text{if } w_{t-1}(\beta) \leq \gamma \\ A_2' X_{t-1}(\beta) + u_t & \text{if } w_{t-1}(\beta) > \gamma \end{cases}$$

[5] This does not imply that price differences lead to shipments in one direction. Recall that the test is applied to the change in the cointegrating residual, which represents movement back to the equilibrating relationship. When the manner in which that movement depends on whether or not the change in the residual is above or below a threshold, shipments could be in either direction, depending on whether the residual is positive or negative, and whether the estimated threshold is positive or negative.

Table 2. Estimates of Cointegration Threshold and Tests of Threshold Cointegration

Commodity Location	Lag	Asymptotic PV	Bootstrap PV	Threshold Estimate	Proportion below Threshold
Corn					
Candor-Williamston	1	0.17	0.16	0.1590	6.2
Cofield-Williamston	1	0.04	0.04	−0.0047	5.4
Kinston-Williamston	2	0.003	0.001	0.0220	34.9
Outlier adjusted data	2	0.116	0.064	0.0259	35.9
Soybeans					
Raleigh-Fayetteville	2	0.004	0.002	−0.0451	5.1
Greenville-Fayetteville	2	0.086	0.078	−0.0397	94.8
Kinston-Fayetteville	2	0.101	0.075	−0.0333	59.2

Note: Lag denotes the lag length in the vector error correction model chosen by the Bayesian Information Criterion. Columns 3 and 4 contain the asymptotic and bootstrapped probability values associated with the test of the null hypothesis of linear cointegration versus the alternative of threshold cointegration. Estimates of the threshold parameter and the proportion of observations lying below the threshold follow.

where γ is the threshold parameter and X_{t-1} is given by

$$(2) \qquad X_{t-1}(\beta) = \begin{bmatrix} 1 \\ w_{t-1}(\beta) \\ \Delta x_{t-1} \\ \Delta x_{t-2} \\ \vdots \\ \Delta x_{t-l} \end{bmatrix}$$

Hansen and Seo provide a quasi-MLE for their threshold cointegration model that involves a nonconventional grid search beginning at a consistent estimate of the cointegrating vector. An evenly spaced grid search over the threshold parameter γ is combined with a grid search over the cointegrating vector, with final estimates taken as those that maximize the log-likelihood. Hansen and Seo assert that it is reasonable to conjecture that conventional standard errors can be reported, but do not provide a formal theory of inference.

A distinguishing feature of the Hansen and Seo specification is the lack of an intercept in the cointegrating relationship. This reduces the dimension of the system and makes it difficult to directly compare findings based on this specification to *all* the results reported by GP, because GP examined both price differentials and the residuals of cointegrating regressions that included an intercept. Extending Hansen and Seo to include a constant in the cointegration relationship would expand its practical application.

Hansen and Seo develop a strategy to test the null hypothesis of linear cointegration against the alternative hypothesis of threshold cointegration using LM test statistics. Parametric bootstrapping is used to estimate asymptotic and small sample probability values. Table 2 contains bootstrap probability values for each bivariate model using the Bayesian Information Criterion to choose the appropriate lag length in the vector error correction model. This table also contains estimates of the thresholds and the proportion of the observations lying below the threshold. With the exception of corn markets in Candor and Williamston, all models indicate threshold cointegration at or about the 10% level of significance. While GP reported similar results using a much different estimation and testing strategy, it appears a single threshold is capable of explaining price behavior in these markets.[6]

Nonlinear Cointegration Tests

Threshold cointegration can be viewed as a special case of a more general nonlinear cointegrating framework. Table 3 presents test results based on Bierens' test. All bivariate pairs appear to be cointegrated. Furthermore, tests of the restrictions on the cointegrating vectors do not reject the null hypothesis that they are $(+1, -1)$ for each combination of variables, with the exception of the Candor and Williamston corn markets. In conjunction with previous results suggesting the Candor and Williamston markets are not threshold-cointegrated, these findings suggest the attractor in these markets may be inherently

[6] This result was robust to the presence of outliers in the Kinston and Williamston corn prices, albeit with somewhat less evidence of threshold cointegration using the outlier-adjusted series.

Table 3. Nonlinear Cointegration Test Results

Commodity Location	Test on Number of Cointegrating Vectors: One vs Two Cointegrating Vectors	Test on Number of Cointegrating Vectors: Zero vs One Cointegrating Vector	Estimate of Cointegrating Vector	Test on Cointegrating Vector Parameters
Corn				
Candor-Williamston	0.282	0.0005	(1.0, −0.745)	2.26
Cofield-Williamston	0.292	0.0002	(1.0, −0.822)	1.81
Kinston-Williamston	0.317	0.0000	(1.0, −0.997)	1.00
Soybeans				
Raleigh-Fayetteville	1.369	0.0000	(−0.9801, 1)	1.31
Greenville-Fayetteville	1.332	0.0000	(−0.9787, 1)	1.23
Kinston-Fayetteville	1.345	0.0000	(−0.9679, 1)	1.26

Note: The test examines the number of cointegrating vectors (r), with critical 5 (10)% values for the test of the null of $r = 1$ vs the alternative of $r = 2$ of 0.054 (0.111). The 5 (10)% value for the test of the null of $r = 0$ versus the alternative of $r = 1$ is 0.017 (0.005). The test on the cointegrating vector examines the null hypothesis that each variable enters with a unit coefficient and has critical values of 4.70, 2.89, and 1.91 at the 5, 10, and 20% levels, respectively.

nonlinear, or appear to be so if transactions costs are time-varying.[7]

Final Remarks

The purpose of this note was to reexamine evidence of threshold cointegration in spatially separated corn and soybean markets. The test results reported here indicate the presence of one threshold in most of the bivariate commodity pairings, and were supported by more general tests of nonlinear cointegration. The exception appeared to be the corn markets between Candor and Williamston, where there was little evidence of threshold cointegration but support for the series following a nonlinear equilibrating relationship, since unit restrictions on the cointegrating vector were rejected. Subsequent work might focus on identifying the extent to which trade flows between these two markets given that they are separated by a mere 50 miles. The findings provide further evidence that transactions costs may create nonlinearities in the relationship between commodity prices across spatially distributed markets.

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[7] The estimated cointegrating vectors are not substantially different from those identified using the Johansen specification. For example, in the Kinston and Williamston markets, the Bierens' vector is estimated to be (1, 0.996) whereas the Johansen estimates are (1, 0.993). For the Johansen specifications, each cointegrating relationship was found to include a constant rather than the error correction model itself, i.e., the restrictions on the intercept of the ECM were not rejected.

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