Dynamics of Commodity Forward Curves [1]

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April 14, 2003

[1]The author wishes to thank, among others, Jari Käppi, Harri Toivonen, and Janne Saarikko for valuable insights in numerous discussions around the subject; and seminar participants at Graduate School of Finance and Financial Accounting in Helsinki.

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Abstract

This paper investigates the factor structure of commodity forward curve dynamics using data from pulp and oil markets. The data used is swap contract quotes, and forward curves are derived from this data using an optimization algorithm. A three factor model explains 89% of the price variation of the oil forward curves and 84% of the price variation of the pulp curves. The factor structure, especially in pulp forward curves, is more complex than found in many other studies, conducted mainly using interest rate data. Possible reasons for this phenomenon are discussed.

Dynamics of the forward curve is important for practitioners pricing and hedging derivatives contracts and for economists studying stochastic movements of economic variables. Traditionally, modelling of the yield curve movements has been an active area of research, both theoretically and empirically. By contrast, research on commodity forward curve dynamics has been relatively scarce. This paper contributes to the research on commodity forward curve dynamics by presenting empirical evidence on the factor structure of long term forward contracts estimated from the broker quotes of the swap contracts.

Analogously to the study of the interest rates, there has been two approaches to the study of commodity prices. The traditional method has focused on modelling the stochastic process for the spot price of the commodity and possibly other state variables, such as convenience yield. Seminal research along these lines include for example Brennan and Schwartz (1985), Gibson and Schwartz (1990) and Schwartz (1997). More recently, the study of the stochastic movements in commodity prices has concentrated on modelling the whole term structure of either forward prices directly Cortazar and Schwartz (1994) or convenience yields, see for example Miltersen and Schwartz (1998). This current paper builds on the whole term structure modelling principle.

Earlier research on the commodity forward curve dynamics has concentrated on modelling the factor structure on the short term contracts, that is, exchange traded futures. Among the first ones along these lines is Cortazar and Schwartz (1994), who studied the dynamics of the futures price curve of copper futures. They found that the factor structure of the copper futures curve was surprisingly similar to the factor structure of yield curve movements. Moreover, the explanatory power of the three two principal components was 97 percent. By contrast, Litterman and Scheinkman (1988) found that the explanatory power of first two principal components of the yield curve movements was remarkably lower, around 90 percent. Clewlow and Strickland (2000) studied the factor structure of NYMEX oil futures and they found that three factors explained over 98 percent of the variation of the futures price movements in the period from 1998 to 2000. A recent paper by Tolmasky and Hindanov (2002) investigated the dynamics of the petroleum futures contracts. In particular, they focused on isolating the effect of seasonality on the factor structure of returns. They found that especially for heating oil, seasonality is an important variable driving the factor structure, however it's statistical significance is somewhat unclear. Crude oil and petroleum markets were not found to be affected by seasonality. In a closely related line of research, Koekebakker and Ollmar (2001) studied the forward curve dynamics using data from the Nord Pool electricity derivatives exchange. The explanatory powers they report are fairly low in comparison, the most likely reason being the extremely complex dynamics of the electricity prices. Koekebakker and Ollmar (2001) use fitted curves as is also done in this current paper.

The results obtained in this paper extend the results of the empirical investigations of the forward curve movements by presenting evidence using data from two distinct commodity markets. In particular, the focus is on modelling the dynamics of the long term forward prices that are implied, not actually observed, from the market quotes of swap contracts. Using the swap market data significantly complicates the analysis and the results obtained are also less clear cut that those obtained using the data on the short term futures contracts. The data used is Brent crude oil swap quotes up to five years, covering the period from 1997 to 2002 and NBSK Risi pulp swap quotes, also up to five years, covering the period from 1998 to 2001. The former data is in the form of monthly observations and the latter is weekly sampled. Principal components analysis (PCA) applied to the implied forward curve movements reveals complex factor structures and the explanatory power of the first three principal components is around 89 percent for oil data and 84 percent for pulp data, respectively.

The paper is organized as follows. Section 1 introduces the underlying theoretical model. Section 2 presents and discusses the data used in the study. Section 3 introduces the method used for estimating the forward curves and Section 4 gives a description of the principal components analysis methodology. In Section 5, the empirical results are presented and discussed. Comparisons with results obtained in earlier studies are taken up. Finally, Section 6 concludes the paper and gives suggestions for further research.

1 Model

The model studied here is similar to the model proposed by Reisman (1991) and Cortazar and Schwartz (1994). The main idea is to model the movements of the forward curve directly, instead of modeling the the forward prices as a function of the spot price process and convenience yield processes. Utilizing this approach, the principal components analysis is applied to the movements of the whole extracted term structure of the forward prices. The procedure for obtaining the term structures of forward prices is explained in the next section.

In the analysis that follows, the spot price of a commodity, at time t, is denoted by $S(t)$. A futures contract at time t, for delivery at time T, is denoted $F(t,T)$. The underlying market is assumed to be complete and frictionless. In the absence of arbitrage opportunities there exists an equivalent martingale measure $\mathbb{Q}$, under which all discounted asset prices are martingales (Harrison and Kreps (1979)).[1] This implies that, under the risk-neutral pricing measure, the instantaneous expected return on all financial assets is the instantaneous risk free rate and hence, the expected return on futures contracts is equal to zero. Therefore, the stochastic process for the futures (and forward) prices is given by

$$dF(t,T) = \sum_{j=1}^{K} \sigma_j(t,T)F(t,T)dW_j(t) \tag{1}$$

and in integrated form

$$F(t,T) = F(0,T)\exp\Big(-\frac{1}{2}\sum_{j=1}^{K}\int_0^t \sigma_j^2(u,T)du + \sum_{j=1}^{K}\int_0^t \sigma_j(u,T)dW_j(u)\Big) \qquad (2)$$

where $dW_1, dW_2, ..., dW_K$ are independent increments of Brownian motions under the risk-neutral measure, and $\sigma_j(t,T)$'s are the volatility functions of the futures (or forward) prices. The volatilities are only functions of time to maturity $(T-t)$. The analysis presented here is based on the assumption of constant interest rates, and therefore, the result by Cox, Ingersoll, and Ross (1981) that under constant interest rates, the futures prices and forward prices are equivalent, can be used.

2 Data

The forward price model (1) describe the stochastic evolution of each of the forward prices on the forward curve under an equivalent martingale measure. However, observations are made under the real world measure. This is not a problem, since only volatility functions are of interest here, and they are invariant with respect to the measure change [2]. The data consists of [observation frequency, katso ed. paperi]. Let $F(t_i, T_j)$ denote the forward price at time t_i, with maturity T_j $(t < T)$, for all $i = 1, 2, ...N$ and $j = 1, 2, ...M$. The instantaneous proportional change in the forward price is approximated by

$$\frac{dF(t_i,T_j)}{F(t_i,T_j)} = \frac{F(t_i,T_j) - F(t_{i-1},T_j)}{F(t_{i-1},T_j)} = x_{i,j} \qquad (3)$$

The data set $\mathbf{X}_{(N,M)}$ is a matrix of returns (3) constructed from the fitted forward price curve and swap price curve

$$\mathbf{X}_{(N,M)} = \begin{pmatrix} x_{1,1} & x_{1,2} & \dots & x_{1,M} \\ x_{2,1} & x_{2,2} & \dots & x_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ x_{N,1} & x_{N,2} & \dots & x_{N,M} \end{pmatrix} \tag{4}$$

The matrix entries are obtained as follows. First, the forward curves are derived from the swap price data using Järvinen (2002). The forward curves are then used to find the individual forward price observations. From these observations, the percentage differences are calculated. This procedure is replicated until the whole matrix is filled. The swap price curve is processed similarly, only that there is, of course, no need for initial derivation of the curve.

The data used in this study consists of weekly NBSK Risi pulp data covering the period from June 1998 to October 2001 and of monthly oil data (European Brent) covering the period from February 1997 to February 2002. The data is obtained from Nordea Bank Finland. The interest rate data used in estimation consists of Libor rates and zero rates. The European interest rate data consists of EuroLibor quotes and zero rates until 1.1.1999 and from that date onwards of Euribor quotes and zero rates. The interest rate data is retrieved from DataStream. The commodity data consists of mid market prices.

3 Curve Estimation

The first step in applying the principal analysis to the forward curve movements, is to estimate the forward curves from the par swap quotes. To introduce the idea, we consider a standard commodity swap contract, with maturity T_N and a fixed contract price $G(t, T_N)$. The present value of this fixed leg of the swap, given the discount bond prices $P(t, T)$ is

$$V_{fx}^N = G(t, T_N) \sum_{i=1}^{N} P(t, T_i) \tag{5}$$

i.e, a simple sum of the present values of the payments. The other side of the swap contract, floating leg, is composed of the present values of the unknown forward prices of the commodity at settlement dates, $i = 1, 2, ...N$. That is,

$$V_{fl}^N = \sum_{i=1}^{N} F(t, T_i) P(t, T_i) \tag{6}$$

where forward prices may be based on average or single observation settlement value. In case we know the forward price curve, the par swap prices for all maturities, $t \geq T$, could be obtained by setting the values of the floating and fixed legs equal. Solving for the par swap price $G(t, T_N)$ gives

$$G(t, T_N) = \frac{\sum_{i=1}^{N} F(t, T_i) P(t, T_i)}{\sum_{i=1}^{N} P(t, T_i)} \tag{7}$$

which shows that the par swap price is defined by the ratio of the discounted value of the floating price payments and an annuity. Therefore, as with interest rate swaps, the par swap price can be interpreted as a present value weighted sum of forward prices.

The method for extracting $F(t, T)$'s is based on the fitting (or optimization) method, introduced by Järvinen (2002). Fitting method is based on the minimization of the sum of squared pricing errors, which is an analogous idea to the option model calibration to observed observed volatilities. The aim is to fit the observed swap prices as precisely as possibly while maintaining a reasonable degree of smoothness in the output forward curve. The target function of the minimization problem is

$$\min_{\{F_i\}_{i=1}^{N}} \left[\sum_{i=1}^{N} \left(V_{fl}(t, T_i) - V_{fx}(t, T_i) \right)^2 + \lambda \sum_{i=1}^{N} \left(F(t, T_i) - F(t, T_{i-1}) \right)^2 \right] \tag{8}$$

where parameter λ is introduced to adjust the weight put on the forward curve smoothness. The forward prices that are the solution of the program are direct forward prices, and not average-based, for details on how this is done, see Järvinen (2002). The optimization problem can be solved using any suitable algorithm. The method utilized in this paper is based on simple gradient search with constant step size and analytic derivatives. For explanation on the various optimization algorithms, see Press, Teukolsky, Vetterling, and Flannery (1992).

In choosing λ, two points need to be mentioned. First, if one picks $\lambda = 0$, then the optimization will converge to the bootstrapped curve and the target function value will be equal to zero. I.e. exact match can be obtained through optimization method. On the other hand, if high enough λ is chosen, then the final forward curve will be flat, starting from the spot value $S(t)$. Obviously, a reasonable value from λ is such that a smooth, yet reasonably shaped and correctly pricing forward curve, will result. For the purposes of this study, λ is set equal to one. The decision is based on experimenting with the algorithm, not explicitly on any objective criteria.

4 Principal Components Analysis

Principal Components Analysis (PCA) is a statistical tool used to identify a structure within a set of interrelated variables. Applying (PCA) to the data, the number of orthogonal factors and the corresponding volatility coefficients, assuming that volatility functions depend only on the time to maturity (T-t)), can be estimated. The data consists of N observations of M different variables, i.e. percentage differences calculated from curve points. Hence, the data matrix is given by (4).

The sample covariance matrix is of order M, and is denoted by ω. The orthogonal decomposition of the covariance matrix is given by

$$\mathbf{\Omega} = \mathbf{C\Lambda C'} \tag{9}$$

where

$$\mathbf{C} = [\mathbf{c_1}, \mathbf{c_2} \ldots \mathbf{c_M}] = \begin{pmatrix} c_{1,1} & c_{1,2} & \ldots & c_{1,M} \\ c_{2,1} & c_{2,2} & \ldots & c_{2,M} \\ \vdots & \vdots & \ddots & \vdots \\ c_{N,1} & c_{N,2} & \ldots & c_{N,M} \end{pmatrix} \tag{10}$$

and

$$\mathbf{\Lambda} = \begin{pmatrix} \lambda_{1,1} & 0 & \ldots & 0 \\ 0 & \lambda_{2,2} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \lambda_{M,M} \end{pmatrix} \tag{11}$$

The $\mathbf{\Lambda}$-matrix is a diagonal matrix and the diagonal elements are the eigenvalues $\lambda_{1,1}, \lambda_{2,2}, ..., \lambda_{M,M}$. $\mathbf{C}$ is an orthogonal matrix of order M. The columns of $\mathbf{C}$ are the eigenvectors corresponding to $\lambda_{j,j}$. $\mathbf{C'}$ denotes the transpose of $\mathbf{C}$. The matrix $\mathbf{P} = \mathbf{XC}$ is the matrix of principal components. The columns $\mathbf{p_j}$ of $\mathbf{P}$ are linear combinations of the columns of $\mathbf{X}$, i.e. the $j : th$ principal component is

$$\mathbf{p_j} = \mathbf{Xc_j} = \mathbf{x_1}c_{1j} + \mathbf{x_2}c_{2j} + \ldots + \mathbf{x_M}c_{Mj} \tag{12}$$

In order to explain all the variance in the sample, one needs to use all M principal components. However, the main idea behind using (PCA) is to reduce the dimensionality of the data. Therefore, the covariance structure is approximated by using $K < M$ largest eigenvalues. The larger the proportion of the explained variance, the better the objective is achieved. The criteria for selecting K is somewhat ambiguous, there exists no clear cut statistical criterion for selection of significant eigenvalues, and hence, the number of factors. [3] Therefore the conventional methodology, employed in the majority of the finance literature, is to add factors until the cumulative explained variance reach a specified limit. This procedure is used also in this study. The proportion of the total sample variance explained can be found using the following formula

$$CEV_K = \frac{\sum_{j=1}^{K} \lambda_j}{\sum_{j=1}^{M} \lambda_M} \tag{13}$$

Where CEV_K denotes *Cumulative explained variance of first K factors.*

5 Empirical Results

Table XX shows the results from the PCA analysis applied to both data sets. Results of the analyses are markedly different, a one factor model is able to explain 62 percent of the variation of returns of the Brent forward prices whereas it can explain only 38 percent of the returns in case of NBSK Risi prices. The explanatory power of one factor model, in particular for NBSK Risi, is very low. The most likely reason for this is the stickiness of the reference index price. NBSK Risi index prices are published only monthly, whereas the swap quotes change in principle daily, and in most cases at least weekly. If more factors are used, then the difference in explanatory power of PCA decreases drastically. A two factor model is able to explain 81 and 63 percent of variation in price returns and a three factor model explains already 89 and 84 percent, respectively. Using the eigenvalue criterion that only eigenvalues that are greater than one are considered significant, the analysis would select a three factor model for Brent and a four factor model for NBSK

Risi. On the other hand, for practical modeling purposes, 90 percent has often been considered a minimal threshold. If this criteria is used, then both data need a four factor model to explain enough variation of returns.

Examination of the factor loadings (figures XX and XY) provides more insight on how the dynamics of the forward curves are determined. The most striking finding of the analysis is that contrary to the factor structure found in many other studies, see for example Litterman and Scheinkman (1988) and Cortazar and Schwartz (1994), there seem not to be obvious level, slope and curvature factors. By contrast, the factors obtained here, exhibit much more complex shapes. There is no general factor that would change the forward prices equally accross maturities. For example, in case of NBSK Risi, a shock to the first factor changes the very short term forward prices upwards (only marginally, however), and the medium term forward prices downwards and finally, the long term forward prices upwards. An interesting note is also that only the first factor is important in explaining the variation in the very long forward prices whereas all four factors have an important impact in explaining the movements in the short end of the forward curve. The conclusion from the the analysis of the factor loadings is that when long term contracts are included in the estimation of the dynamics of the forward curve, the complexity of curve changes increases dramatically. Possible answer to these results can possibly found from the strongly mean-reverting nature of the commodity prices and complex interplay between future demand and supply expectations and possiblity of storing these commodities for speculating on or hedging. Another possibility is that there are great differences in the liquidity of the contracts of different maturities, and in particular, that the reference index value changes non-synchronously with the prices of the swap contracts. It is likely that both these factors have an effect on the estimation results.

6 Conclusions

This paper has investigated the dynamics of the commodity forward price curves using the principal components analysis on the Brent oil and NBSK Risi pulp data. The forward curves have been estimated using the fitting method, presented in Järvinen (2002). The data is in the form of swap quotes from 1998 to 2002 for Brent and from 1997 to 2002 for NBSK Risi. The longest maturity swaps are five years. The principal component analysis was conducted on the percentage changes of the forward prices from the fitted forward curves.

The main findings of the study are: At least three and even four factors are needed in order to adequately model the dynamics of the forward curves. This is true for both data series. Interestingly, the factor structures of the markets analyzed are markedly different, and furthermore, do not resemble the structures found in earliers studies. Namely, level, slope and curvature. By contrast, the results derived in this paper reveal that the short end of the curve is very demanding from the modeling point of view. The long end of the curve, on the other hand, turned out to exhibit simpler dynamics. Possible reasons for these findings include: Complex demand, supply and storage dynamics leading to non-synchronously moving forward prices, in particular, in the short end of the curve. Liquidity and reliability of the swap quotes from which the forward curves are derived, and stickiness of the reference index quotes that provide the pseudo spot index value for the fitting algorithm. Finally, the results are naturally dependent on the forward curve estimation algorithm. Since the forward curve points cannot be estimated uniquely from the swap data, a fitting method was necessary in order to obtain the forward curves.

This paper provides first documented results from application of PCA to model the dynamics of the commodity forward curves using long term swap contract data. Earlier studies have used readily available futures data. As the OTC market for long term commodity derivative contracts is growing rapidly, more research is needed in order to investigate the statistical behavior of the prices of these contracts. As new data arrives at an expanding rate, this provides academic community fruitful opportunities to investigate the subject further.

Notes

[1]The equivalent martingale measure is commonly called the risk-neutral measure, in particular in the applied research.

[2]This is strictly correct only in continuous time

[3]Criteria discussed in the literature includes: 1) Scree plot test; the test is carried out by graphical inspection of the eigenvalue plot. Eigenvalues are added until the plot levels off. 2) Eigenvalue criterion; eigenvalues that are greater than one are considered significant.

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Table 1: NBSK Risi Forwards Summary Statistics

Statistic	Index	6m	1y	2y	3y	5y
Mean	-0.0009	-0.0004	-0.0004	-0.0004	-0.0004	0.0000
Standard Deviation	0.0167	0.0109	0.0116	0.0098	0.0109	0.0184
Kurtosis	9.2818	1.9003	2.2834	1.7928	1.5060	4.7249
Skewness	-0.4417	0.6824	0.4424	0.0452	0.3460	0.2415
Range	0.1495	0.0688	0.0778	0.0646	0.0656	0.1514
Minimum	-0.0755	-0.0306	-0.0327	-0.0324	-0.0305	-0.0555
Maximum	0.0741	0.0382	0.0451	0.0323	0.0351	0.0958
Count	177	177	177	177	177	177

Table 2: NBSK Risi Swaps Summary Statistics

Statistic	Index	6m	1y	2y	3y	5y
Mean	-0.0009	-0.0006	-0.0005	-0.0005	-0.0005	-0.0004
Standard Deviation	0.0167	0.0099	0.0101	0.0080	0.0073	0.0059
Kurtosis	9.2818	1.8828	2.4779	1.7625	0.9910	2.2131
Skewness	-0.4417	-0.2175	0.4850	0.0794	-0.0438	0.3648
Range	0.1495	0.0626	0.0672	0.0537	0.0382	0.0343
Minimum	-0.0755	-0.0333	-0.0289	-0.0279	-0.0188	-0.0169
Maximum	0.0741	0.0293	0.0383	0.0258	0.0194	0.0175
Count	177	177	177	177	177	177

Table 3: Brent Forwards Summary Statistics

Statistic	Index	6m	1y	2y	3y	5y
Mean	0.0062	0.0024	0.0021	0.0051	0.0025	0.0008
Standard Deviation	0.1255	0.0650	0.0578	0.0547	0.0491	0.0630
Kurtosis	0.5542	0.4751	1.7357	1.3674	0.4552	0.5037
Skewness	0.3486	-0.1424	0.6150	0.5294	0.1060	0.5465
Range	0.6924	0.3386	0.3400	0.2950	0.2471	0.3165
Minimum	-0.3059	-0.1754	-0.1299	-0.1239	-0.1235	-0.1340
Maximum	0.3866	0.1631	0.2101	0.1712	0.1236	0.1825
Count	62	62	62	62	62	62

Table 4: Brent Swaps Summary Statistics

Statistic	Index	6m	1y	2y	3y	5y
Mean	0.0062	0.0031	0.0018	0.0023	0.0024	0.0016
Standard Deviation	0.1255	0.0709	0.0645	0.0434	0.0383	0.0390
Kurtosis	0.5542	1.3295	1.0097	0.3944	0.3283	0.4677
Skewness	0.3486	0.2551	-0.4186	0.0352	0.0610	0.1655
Range	0.6924	0.4248	0.3501	0.2161	0.1887	0.1889
Minimum	-0.3059	-0.1959	-0.2063	-0.1099	-0.0846	-0.0859
Maximum	0.3866	0.2290	0.1439	0.1062	0.1041	0.1030
Count	62	62	62	62	62	62

Table 5: NBSK Risi Fwds Correlations

	Index	6y	1y	2y	3y	5y
Index	1	-0.01	-0.09	-0.06	-0.06	0.02
6m	-0.01	1	0.85	0.06	-0.01	0.01
1y	-0.09	0.85	1	0.17	-0.05	-0.12
2y	-0.06	0.06	0.17	1	0.64	-0.36
3y	-0.06	-0.01	-0.05	0.64	1	-0.03
5y	0.02	0.01	-0.12	-0.36	-0.03	1

Table 6: NBSK Risi Swap Correlations

	Index	6m	1y	2y	3y	5y
Index	1	0.48	0.23	0.15	0.07	0.07
6m	0.48	1	0.62	0.62	0.4	0.47
1y	0.23	0.62	1	0.83	0.6	0.51
2y	0.15	0.62	0.83	1	0.73	0.6
3y	0.07	0.4	0.6	0.73	1	0.67
5y	0.07	0.47	0.51	0.6	0.67	1

Table 7: Brent Fwd Correlations

	Index	6m	1y	2y	3y	5y
Index	1	0.36	0.14	-0.25	-0.13	-0.13
6m	0.36	1	0.84	-0.03	0.08	-0.02
1y	0.14	0.84	1	0.26	0.33	0.17
2y	-0.25	-0.03	0.26	1	0.86	0.71
3y	-0.13	0.08	0.33	0.86	1	0.91
5y	-0.13	-0.02	0.17	0.71	0.91	1

Table 8: Brent Swap Correlations

	Index	6m	1y	2y	3y	5y
Index	1	0.78	0.63	0.47	0.3	0.16
6m	0.78	1	0.87	0.74	0.53	0.33
1y	0.63	0.87	1	0.88	0.68	0.46
2y	0.47	0.74	0.88	1	0.93	0.77
3y	0.3	0.53	0.68	0.93	1	0.93
5y	0.16	0.33	0.46	0.77	0.93	1

Table 9: NBSK Risi Fwd Eigenvalues

Eigenval	Variance (% total)	Cumul Eigenval	Cumul %
7.610	38.051	7.610	38.051
5.001	25.006	12.611	63.057
4.278	21.390	16.889	84.447
1.656	8.280	18.545	92.727
0.669	3.346	19.215	96.073
0.108	0.539	19.322	96.612
0.058	0.289	19.380	96.901
0.048	0.240	19.428	97.141
0.048	0.240	19.476	97.380
0.048	0.238	19.524	97.619

Table 10: Brent Fwd Eigenvalues

Eigenval	Variance (% total)	Cumul Eigenval	Cumul %
12.378	61.888	12.378	61.888
3.842	19.210	16.220	81.098
1.627	8.135	17.847	89.233
0.932	4.659	18.778	93.892
0.330	1.651	19.108	95.542
0.217	1.083	19.325	96.625
0.053	0.266	19.378	96.891
0.050	0.248	19.428	97.139
0.048	0.241	19.476	97.380
0.048	0.239	19.524	97.619

Table 11: NBSK Risi Fwd Factor Loadings

	Factor 1	Factor 2	Factor 3	Factor 4
3 month	0.1286	0.4087	-0.4804	-0.3098
6 month	0.0461	0.5391	-0.6561	-0.4472
9 month	-0.0444	0.5104	-0.6815	-0.4333
12 month	-0.0992	0.5437	-0.7116	-0.2477
15 month	-0.1437	0.6316	-0.6914	0.1920
18 month	-0.1907	0.6210	-0.4666	0.5726
21 month	-0.3033	0.6493	-0.1704	0.6459
24 month	-0.4914	0.7227	0.2408	0.3678
27 month	-0.5327	0.6310	0.5266	-0.0289
30 month	-0.4893	0.5516	0.6181	-0.1880
33 month	-0.4078	0.5487	0.6616	-0.2374
36 month	-0.1616	0.6409	0.6822	-0.2434
39 month	0.4938	0.6775	0.4853	-0.1397
42 month	0.8639	0.4303	0.1604	0.0042
45 month	0.9337	0.2831	0.0601	0.0524
48 month	0.9520	0.2124	0.0559	0.0628
51 month	0.9592	0.1691	0.0739	0.0635
54 month	0.9624	0.1388	0.0891	0.0630
57 month	0.9639	0.1200	0.0967	0.0630
60 month	0.9643	0.1135	0.0987	0.0632
Expl.Var	7.6102	5.0011	4.2780	1.6561
Prp.Totl	0.3805	0.2501	0.2139	0.0828

Table 12: Brent Fwd Factor Loadings

	Factor 1	Factor 2	Factor 3	Factor 4
3 month	-0.1198	-0.5095	0.5732	-0.5889
6 month	-0.0133	-0.8371	0.3798	-0.3023
9 month	0.1552	-0.9329	0.0182	0.1437
12 month	0.2625	-0.8840	-0.1496	0.3018
15 month	0.4091	-0.8136	-0.2359	0.2296
18 month	0.6779	-0.5946	-0.3486	-0.0340
21 month	0.8166	-0.1628	-0.4142	-0.2615
24 month	0.8263	0.0900	-0.3969	-0.2973
27 month	0.8764	0.1521	-0.3237	-0.2454
30 month	0.9237	0.1429	-0.1991	-0.1520
33 month	0.9266	0.1518	-0.0978	-0.0732
36 month	0.9329	0.1509	-0.0256	-0.0258
39 month	0.9578	0.0879	0.0582	0.0076
42 month	0.9644	-0.0006	0.1500	0.0383
45 month	0.9511	-0.0266	0.2126	0.0646
48 month	0.9389	0.0060	0.2484	0.0873
51 month	0.9280	0.0636	0.2702	0.1076
54 month	0.9153	0.1209	0.2859	0.1252
57 month	0.9023	0.1649	0.2982	0.1391
60 month	0.8926	0.1905	0.3068	0.1480
Expl.Var	12.3776	3.8420	1.6270	0.9317
Prp.Totl	0.6189	0.1921	0.0814	0.0466

Figure 1: Plot of Eigenvalues (NBSK Risi Fwd)

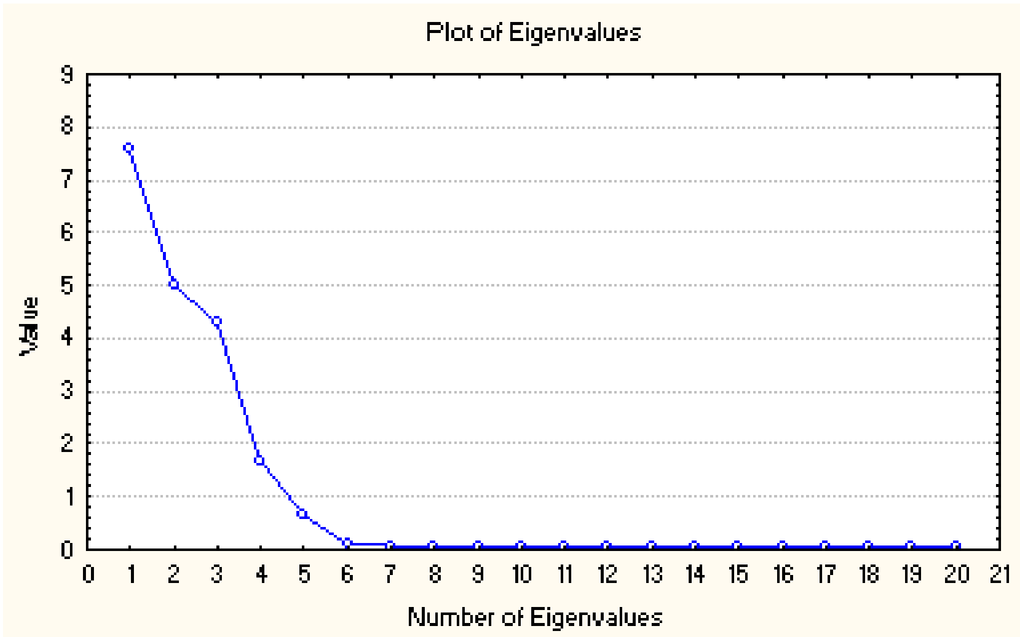

Figure 2: Plot of Eigenvalues (Brent Fwd)

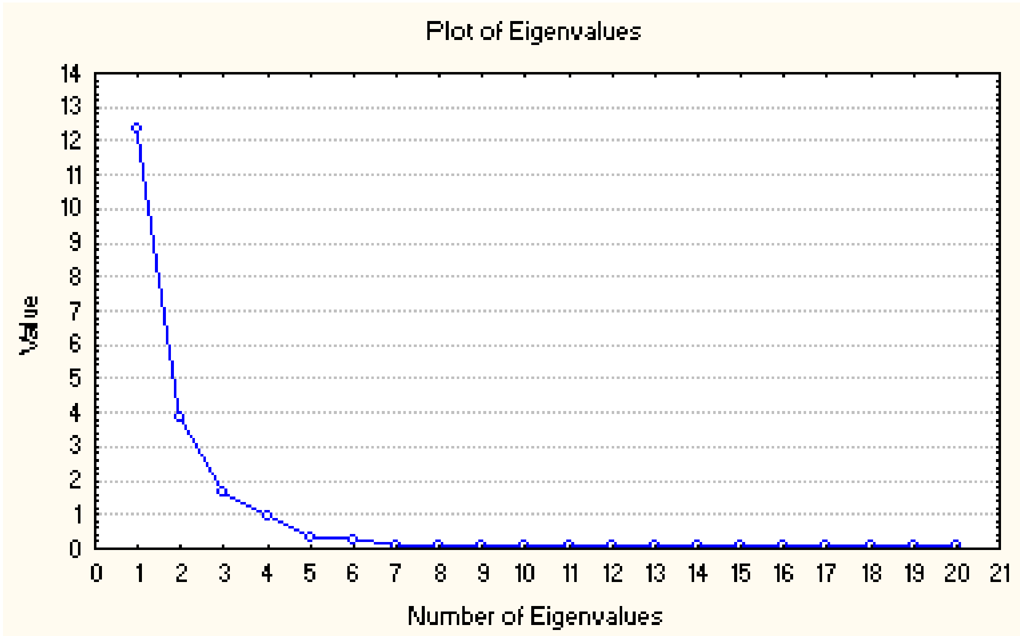

Figure 3: Plot of Factor Loadings (NBSK Risi Fwd)

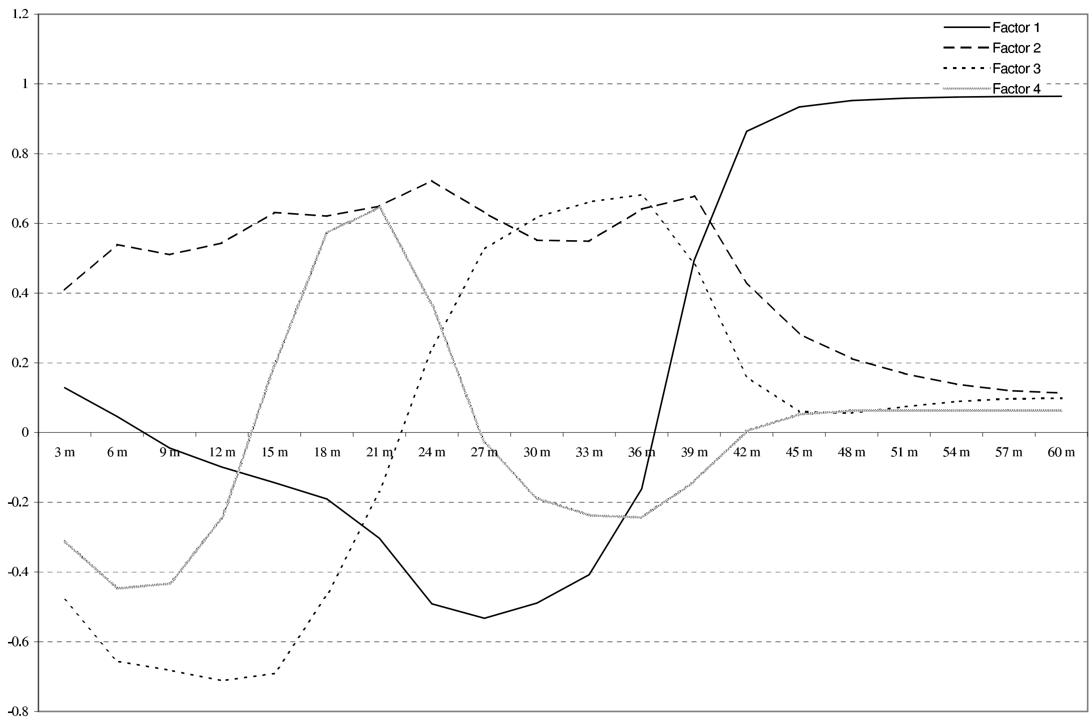

Figure 4: Plot of Factor Loadings (Brent Fwd)